UNIT-1 Antenna Basics

Introduction: Antennas have become important part in our day-to-day life. They are everywhere: at our homes and work places, on our cars and air craft, while our ships, Satellites and Space Crafts. Even as pedestrians, we carry them.

Antennas may be of infinite varieties. But all operate according to the same basic principle of electromagnetic. Antennas are our electronic eyes and ears on the world. They are an essential and integral part of our civilization.

Basic radiation equation: Antenna may be defined as a structure associated with a region of transition between a guided wave and free space wave or vice-versa. Antennas convert electrons to photons or vice-versa. Regardless of antenna type, all involve the same basic principle that radiation is produced by accelerated charge. The basic equation of radiation may be expressed simply as

$$\dot{I}L = Q\dot{v} \qquad \text{Basic radiation equation}$$

Where $\dot{I}$ = time changing current, As^{-1}

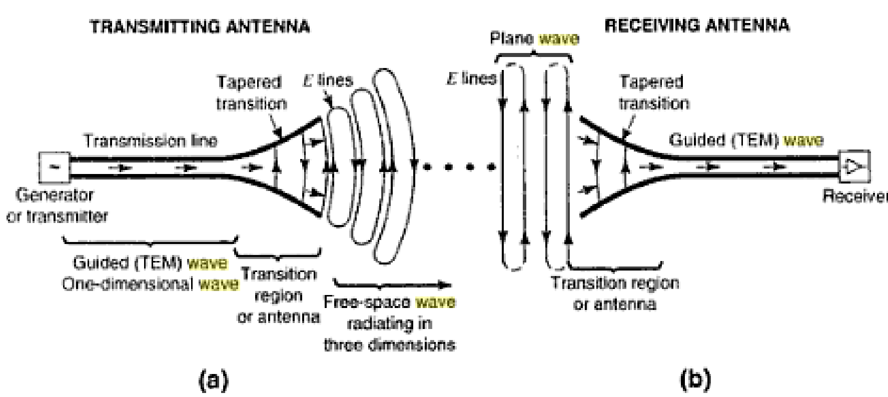
L= Length of current element, m

Q= Charge, C

$\dot{v}$= time changing velocity equals the acceleration of the charge, ms^{-2}

Thus, time changing current radiates and accelerated charge also radiates. The radiation is perpendicular to the square of $\dot{I}L \text{ or } Q\dot{v}$.

Two wire Transmission line as an Antenna:



The two wire transmission line in figure (a) is connected to a radio frequency generator or transmitter. Along the uniform part of the line, energy is guided as a plane transverse electromagnetic mode (TEM) wave with little loss. The spacing between wires is assumed to be small fraction of a wave length. Further on the transmission line opens out in a tapered transition. As the separation approaches the order of wavelength or more, the wave tends to be radiated so that the opened out line acts like an antenna which launches a free space wave. The currents on the transmission line flow out on the antenna and end there, but the fields associated with them keep on going.

The transmitting antenna in figure (a) is a region of transition from a guided wave on a transmission line to a free space wave. The receiving antenna in figure (b) is a region of transition from a free space wave to guided wave on a transmission line. Thus an antenna is a transition device, or transducer, between a guided wave and free space wave or vice-versa. The antenna is a device which interfaces a circuit and space.

From the circuit point of view, the antennas appear to the transmission line as a resistance R_r, called the radiation resistance. It is not related to any resistance in the antenna itself. But is a resistance coupled from space to the antenna terminals.

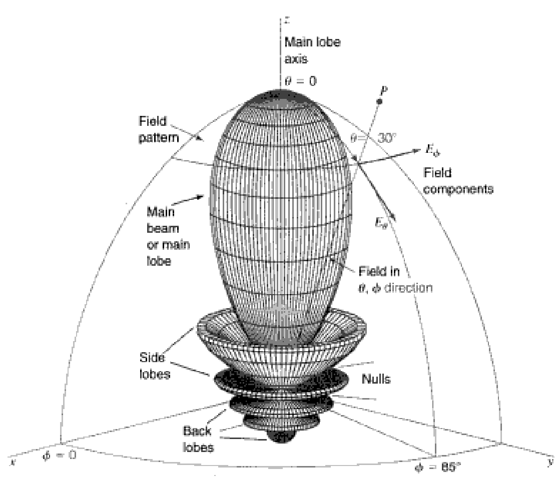
Patterns: The radiation resistance is the variation of field or power (proportional to the field squared) as a function of the spherical coordinates θ and φ. So radiation patterns are 3-dimensional quantities.

To completely specify the radiation pattern with respect to the field intensity and polarization requires three patterns. They are

(1). The θ component of the electric field as a function of the angles θ and φ or $\mathbf{E_\theta(\theta,\varphi)}$ measured in vm^{-1}.

(2). The φ component of the electric field as a function of the angles θ and φ or $\mathbf{E_\varphi (\theta,\varphi)}$ measured in vm^{-1}.

(3). The phases of these fields as a function of the angles θ and φ or $\mathbf{\delta_\theta(\theta,\varphi)}$ and $\mathbf{\delta_\varphi (\theta,\varphi)}$ measured in radians or degrees.



Any field pattern can be represented in 3-dimensional spherical coordinates as shown in above figure which shows the 3-dimensional field pattern with pattern radius, r (from origin to pattern boundary at the dot) proportional to the field intensity in the direction θ and φ. The pattern has its main lobe (beam) or maximum radiation in the z-direction (θ=0) with minor lobes (side and back) in other directions.

Normalized field and power patterns: Dividing a field component by its maximum value, we obtain a normalized or relative field pattern which is a dimension less number with maximum value of unity. Thus the normalized pattern for the electric field is given by

Normalized field pattern= $E_\theta(\theta,\varphi)_n = E_\theta(\theta,\varphi) / E_\theta(\theta,\varphi)_{max.}$ **(Dimension less)**

Patterns may also be expressed in terms of the power per unit area (or pointing vector $S(\theta,\varphi)$).Normalizing this power with maximum value of unity. Thus the normalized power pattern is given by

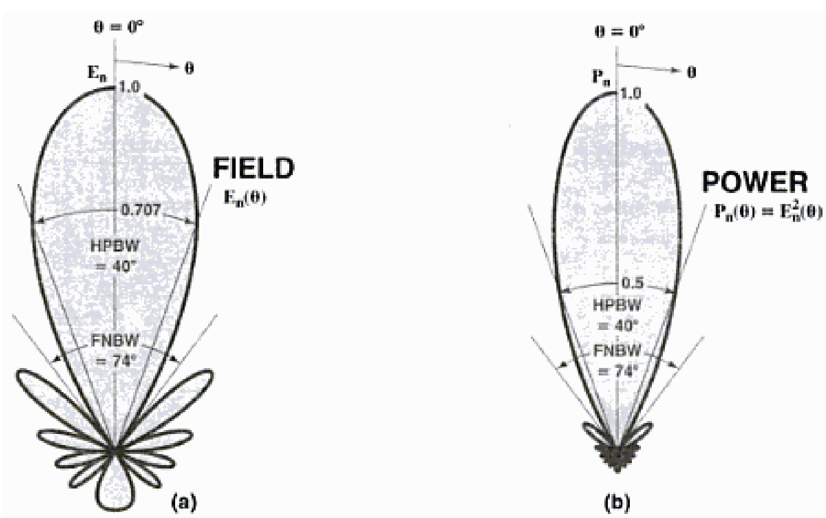
Normalized power pattern= $P_n(\theta,\varphi)_n = S(\theta,\varphi) / S(\theta,\varphi)_{max.}$ **(Dimension less)**

Where : $S(\theta,\varphi)$=Poynting vector = $[E_\theta^2(\theta,\varphi) + E_\varphi^2(\theta,\varphi)]/Z_0$ Wm^{-2}

$S(\theta,\varphi)_{max.}$= maximum value of $S(\theta,\varphi)$ Wm^{-2}

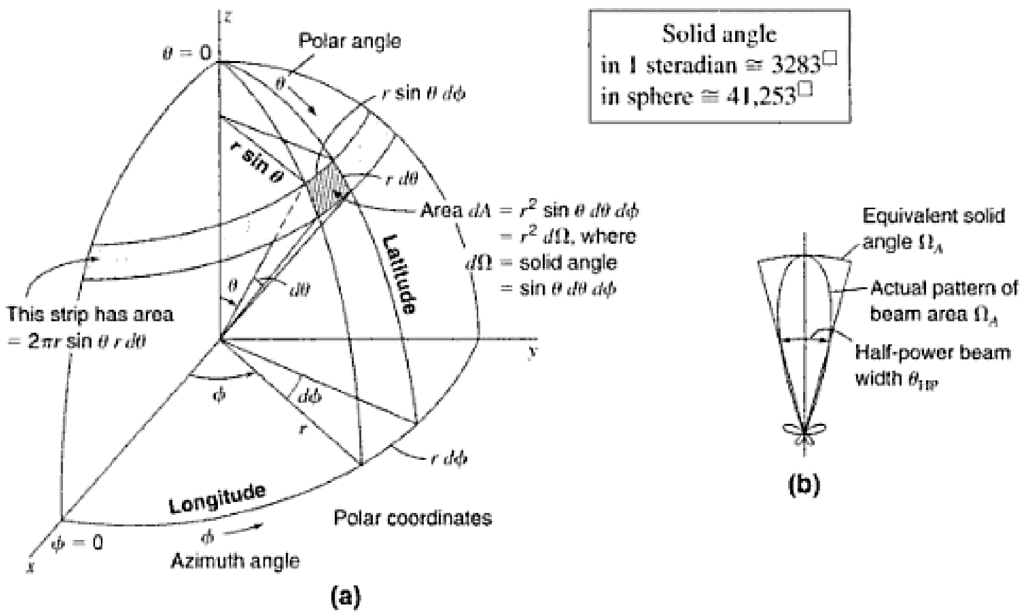
Z_0= intrinsic impedance of free space = 377Ω.

HPBW and FNBW: Any field pattern can be represented in 3-dimensional spherical coordinates as shown in the above figure or by plane cuts through the main lobe axis. Two such cuts at right angles called the principal plane patterns in xz and yz planes may be required but if the pattern is symmetrical around the z-axis, one cut is sufficient. The below figures (a) and (b) are the principal plane field and power patterns in polar coordinates.



The angular beam width at the half power level or half power beam width (HPBW) or 3-Db beam width and the beam width between first nulls (FNBW) are also shown in figures. These two are important pattern parameters.

Beam area or beam solid angle (Ω_A): In polar two dimensional coordinates an incremental area dA on the surface of a sphere is the product of the length rdθ in the θ direction and rsinθdφ in the φ direction as shown in below figure.



From the figure dA= (rdθ) (rsinθdφ) = r^2 dΩ

Where dΩ= sinθdθdφ=solid angle subtended by the area dA (in steradians or square degrees).

The area of the strip of width rdθ extending around the sphere at a constant angle θ is given by (2πrsinθdθ) (rdθ). Integrating this for θ values from 0 to π yields the area of the sphere. Thus,

Area of the sphere=$2\pi r^2$ sinθdθ=$2\pi r^2$[-cosθ] = $2\pi r^2$[-(1)-(-1)] = $4\pi r^2$

where 4π = solid angle subtended by a sphere, Sr. Thus one steradian=(solid angle of sphere)/4πor 1 rad^2=$(180/\pi)^2$ $(deg)^2$ =3282.8064 sq. degrees = solid angle in sphere.

The beam area or beam solid angle or Ω_A of an antenna is given by the integral of the normalized power pattern over a sphere (4πSr). That is Ω_A= Pn(θ,φ) sinθdθdφ or

Ω_A= P_n(θ,φ) dΩ (Sr). The beam area of an antenna can often be described approximately in terms of the angles subtended by the half power points of the main lobe. Thus

Beam area~ $\Omega_A \sim \Theta_{HP}\, \varphi_{HP}$ (Sr) where Θ_{HP} and φ_{HP} are the half power beam widths along θ and φ directions respectively.

Radiation Intensity: The power radiated from an antenna per unit solid angle is called the radiation intensity, U (θ,φ) measured in watts per steradian or watts per square degrees. The normalized power pattern can be expressed in terms of U (θ,φ) as $P_n(\theta,\varphi) = U(\theta,\varphi) / U(\theta,\varphi)_{max}$. Hence the pointing vector, S depends on the distance from the antenna but, the radiation intensity U is independent of the distance.

Beam efficiency: The total beam area Ω_A consists of the main beam area Ω_M plus the minor lobe area Ω_m.i.e. $\Omega_A = \Omega_M + \Omega_m$.

The ratio of the main beam area to the total beam area is called the beam efficiency, ϵ_M. Thus

Beam efficiency = $\epsilon_M = \Omega_M / \Omega_A$.(dimension less)

The ratio of minor lobe area (Ω_m) to the total beam area is called the stray factor (ϵ_m) **i.e.** $\epsilon_M = \Omega_m / \Omega_A$. It follows that $\epsilon_M + \epsilon_m = 1$.

Directivity, D and gain, G: The directivity of an antenna is equal to the ratio of the maximum power density over a sphere $P(\theta,\varphi)_{max}$. to it's average value over a sphere. Thus

D = $P(\theta,\varphi)_{max} / P(\theta,\varphi)_{ave}$.

The directivity is a dimensionless ratio greater than or equal to 1always. The average power over a sphere is given by

$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}} \qquad \textit{Directivity from pattern}$$

The directivity is a dimensionless ratio ≥1.

The average power density over a sphere is given by

$$P(\theta,\phi)_{av} = \frac{1}{4\pi}\int_{\phi=0}^{\phi=2\pi}\int_{\theta=0}^{\theta=\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi = \frac{1}{4\pi}\iint_{4\pi} P(\theta,\phi)\, d\Omega \qquad (\mathrm{W\ sr^{-1}})$$

Therefore, the directivity

$$D = \frac{P(\theta,\phi)_{max}}{(1/4\pi)\iint_{4\pi} P(\theta,\phi)\, d\Omega} = \frac{1}{(1/4\pi)\iint_{4\pi}[P(\theta,\phi)/P(\theta,\phi)_{max}]\, d\Omega}$$

and

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A} \qquad \textit{Directivity from beam area } \Omega_A$$

where $P_n(\theta,\phi)\, d\Omega = P(\theta,\phi)/P(\theta,\phi)_{max}$ = normalized power pattern

Smaller the beam area, The larger is the directivity D.

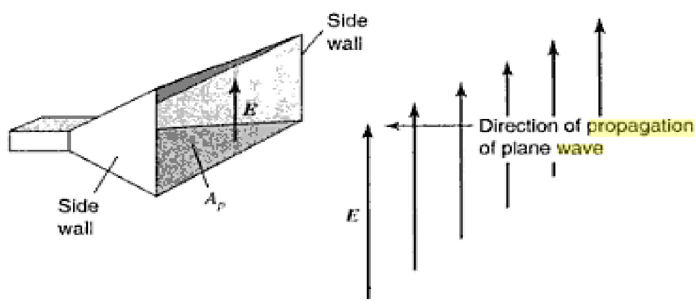
The gain G of an antenna is an actual or realized quantity which is less than the directivity D due to ohmic losses in the antenna. The ratio of gain to directivity is the antenna efficiency factor. Thus G= k D Where k is the efficiency factor (between 0 and1) and is dimension less.

Directivity and resolution (N): The resolution of an antenna may be defined as equal to half the beam width between first nulls i.e. (FNBW)/2. Half the beam width between first nulls is approximately equal to the half power beam width i.e. (FNBW)/2 ~ HPBW. Thus the product of the (FNBW)/2 in the two principal planes of the antenna pattern is a measure of the antenna beam area i.e.

$$\Omega_A = \left(\frac{\text{FNBW}}{2}\right)_\theta \left(\frac{\text{FNBW}}{2}\right)_\phi$$

Then the resolution (equal to directivity) N is given by N=4π/ Ω_A. i.e. we may conclude that ideally the number of point sources an antenna can resolve is numerically equal to the directivity of the antenna i.e. D =N.

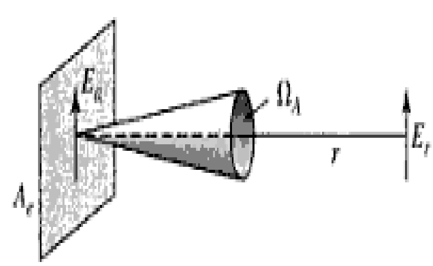
Antenna apertures: Consider a receiving antenna which is a rectangular electromagnetic horn immersed in the field of a uniform plane waves as shown in below figure.



Let the pointing vector or power density of the plane wave be S watts per square meter and the area or physical aperture of the horn be A_p square meters. If the horn extracts all the power from the wave over it's entire physical aperture, then the total power P absorbed from the wave is $P=(E^2/Z)A_p = S\ A_p$ (watts).

But the field response of the horn is not uniform across the aperture A because E at the side walls must equals zero. Thus the effective aperture A_e of the horn is less than physical aperture A_p as given by $\epsilon_{ap} = A_e/A_p$ where ϵ_{ap} is the aperture efficiency.

Consider now an antenna with an effective aperture Ae, which radiates all of its power in a conical pattern of beam area Ω_A as shown in below figure.



Assuming a uniform field E_a over the aperture, the power radiated is P= $(E_a^2/Z_0)\ A_e$ (watts) where Z_0 is insintric impedance of medium (377Ω for free space). Assuming a uniform field $\mathbf{E}_r$ at a distance r, the power radiated is also given by P=$(E_r^2/Z_0)\ r^2\ \Omega_A$ (watts). Equating the above two equations yields $(E_a^2/Z_0)\ A_e = (E_r^2/Z_0)\ r^2\ \Omega_A$

$$E_r^2\ r^2\ \Omega_A = E_a^2 A_e$$
$$E_r^2 = (E_a^2 A_e)/(r^2\ \Omega_A)$$
$$E_r = (E_a \sqrt{A_e})/(r\sqrt{\Omega_A})$$
$$E_r = (E_a A_e)/(r\sqrt{A_e \Omega_A})$$
$$E_r = E_a A_e/(r\lambda)$$

Where $\lambda = \sqrt{A_e \Omega_A}$

$$\lambda^2 = A_e \Omega_A$$

Thus if A_e is known, we can determine Ω_A at given wave length. From this relation we can write the directivity D as $D=4\pi/\Omega_A=4\pi/(\lambda^2/A_e) = 4\pi A_e/\lambda^2$.

Directivity at glance: The different formulas from different parameters of antenna are given by

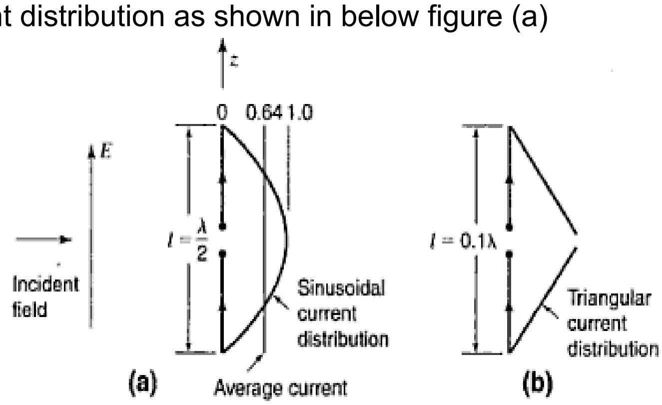
$$D = \frac{P(\theta,\phi)_{max}}{P(\theta,\phi)_{av}} \qquad \textit{Directivity from pattern}$$

$$D = \frac{4\pi}{\iint_{4\pi} P_n(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_A} \qquad \textit{Directivity from beam area } \Omega_A$$

$$D = N$$

$$D = 4\pi \frac{A_e}{\lambda^2} \qquad \text{(dimensionless)} \qquad \textit{Directivity from aperture}$$

Effective Height: The effective height (he) may be defined as the ratio of the induced voltage to the incident field i.e. he=v/**E** (m). The effective height can also be defined from the current distribution. In this case it is defined as the average value of current distribution multiplied by its physical height. For example if a dipole antenna of height l=λ/2 is having a sinusoidal current distribution, then its effective height is given by he=0.64l. Where 0.64 is the average value of the sinusoidal current distribution as shown in below figure (a)



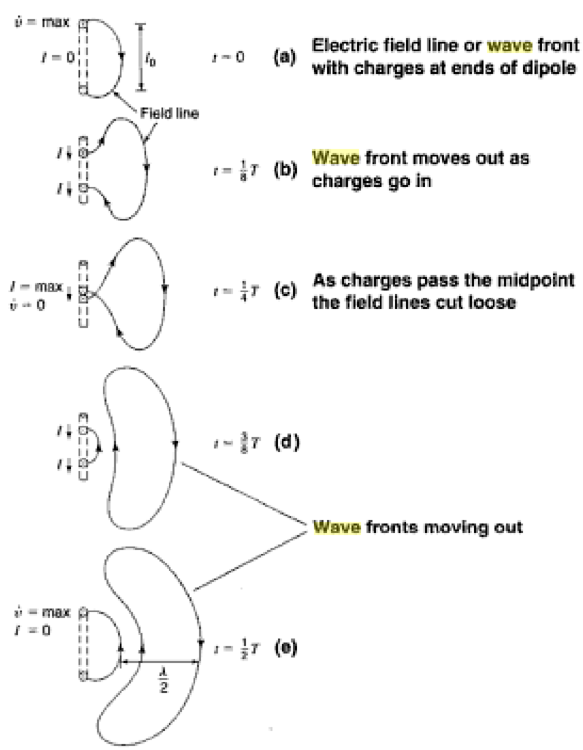
Similarly if another dipole antenna of physical length l= λ/10 is having a triangular current distribution as shown in figure (b). Then the effective length or height is given by le (or he)= 0.5l. where 0.5 is the average value of the triangular current distribution. The another way of defining effective height is to consider the transmitting antenna case and equating the effective height to the physical height multiplied by the average current i.e.

$$h_e = \frac{1}{I_0}\int_0^{h_p} I(z)\,dz = \frac{I_{av}}{I_0}h_p \qquad \text{(m)}$$

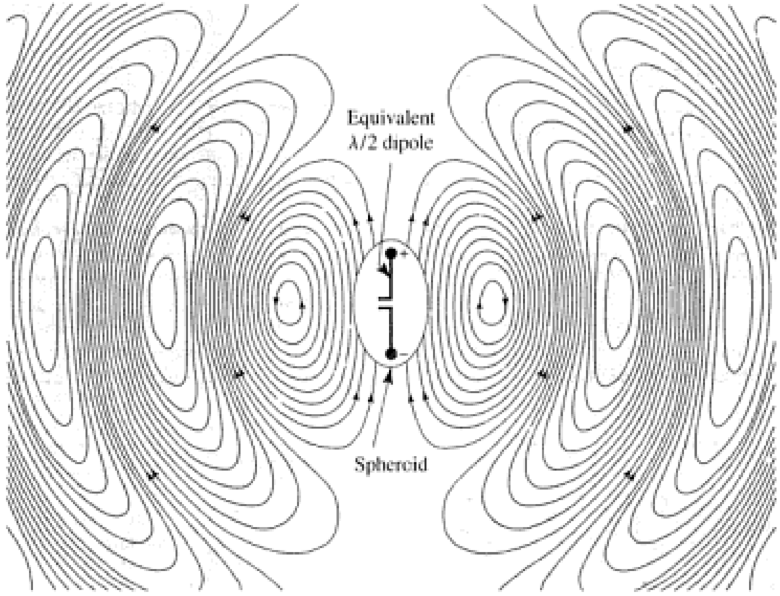
where

h_e = effective height, m
h_p = physical height, m
I_{av} = average current, A

Fields from Oscillating Dipole: Consider a dipole antenna which has two equal charges of opposite sign oscillating up and down in harmonic motion as shown in below figure.

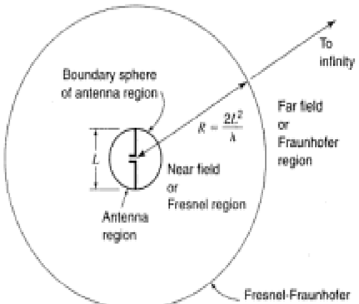


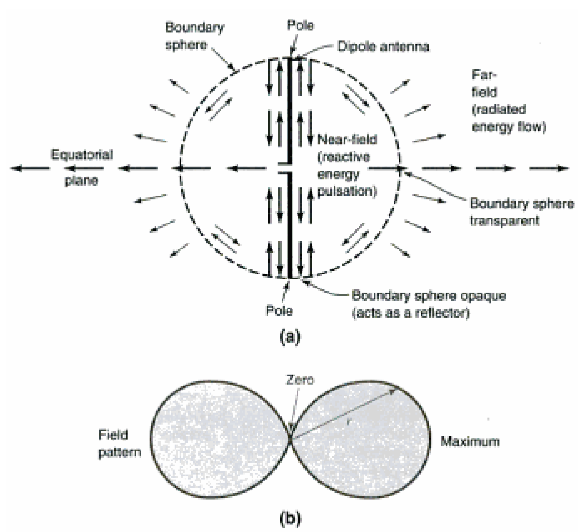
Here we are concentrating on how the electric field is produced. For clarity in understanding we are drawing a single electric field line. At time t=0, the charges are at maximum separation and undergo acceleration as shown in figure (a). At 1/8 period later, the charges are moving towards each other and electric field line will be as shown in figure(b). And at a ¼ period they pass at the midpoint as shown in figure(c). As this happens the field lines detach and new ones of opposite sign are formed. As time progresses to a ½ period the field lines continue to move as shown in figure (d) and (e). An oscillating dipole with more field lines is shown in below figure.



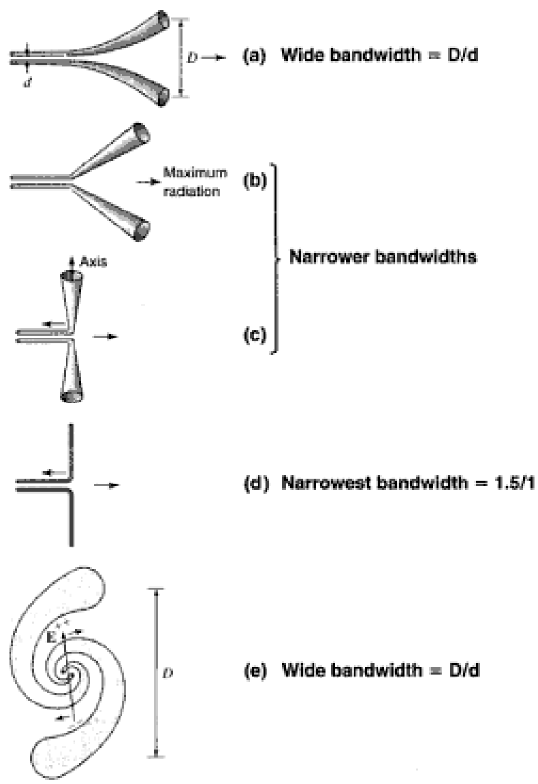
Antenna field Zones: The fields around an antenna may be divided into two regions or zones. One near the antenna called the near field or Fresnel zone and one at a larger distance called the far field or Fraunhofer zone. The boundary between the two may be arbitrarily taken to be a radius R= $2L^2/\lambda$ (meters). Where L is length of antenna and λ is wavelength.

In the far or Fraunhofer region, the measurable field components are perpendicular (transverse) to the radial direction from the antenna and all power flow is directed radially outward. In the far field the shape of field pattern is independent of distance. In near or Fresnel region, The longitudinal component of the electric field may be significant and power flows is not entirely radial. In the near field, the shape of the field pattern depends on the distance as shown in below figures.





Shape-Impedance Considerations: It is possible in many cases to deduce the qualitative behavior of an antenna from its shape. It is explained in below figures. Starting with the opened out two conductor transmission line of figure (a).



In figure (b) curved conductors are straightened into regular cones and in figure (c) the cones are aligned collinearly forming a bi conical antenna. In figure (d) the cones degenerate into straight wires. In figure (e) the two conductors are curved more sharply and in opposite directions resulting in spiral antenna. Going from figures (a) to (e) in (a) and (b) the field will be unidirectional and in figures (c) and (d) the field will be bidirectional whereas in (e) the field is omnidirectional i.e. the field will radiate in all directions.

Radiation Resistance (R_r): It is a part of antennas feed point resistance that is caused by the radiation of electromagnetic waves from the antenna. Physically it is not available so it is virtual resistance represented as R_r. So the radiation resistance of an antenna is a good indicator of the strength of the electromagnetic field radiated by a transmitting antenna or being received by a receiving antenna.

Antenna Temperature (T_A): Every object with a physical temperature above absolute zero radiates energy. The amount of energy radiated is usually represented by an equivalent temperature T_B known as brightness temperature which is defined as

$T_B(\theta,\varphi) = \epsilon(\theta,\varphi)T_m$. where T_B is brightness temperature (0deg.Kelvin), ϵ is emissivity (dimensionless) and T_m is molecular(physical) temperature (0 deg Kelvin).

The brightness temperature emitted by the different sources is intercepted by antennas and it appears at the terminals as antenna temperature. That is considering the gain of the antenna into account the antenna temperature is defined as:

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta, \phi)\, G(\theta, \phi) \sin\theta \, d\theta \, d\phi}{\int_0^{2\pi}\int_0^{\pi} G(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$

Where TA= antenna temperature (Kelvin)

G(θ,φ) = gain of the antenna.

Front-to-Back ratio (FBR): The ratio of the signal strength transmitted in a forward direction to that transmitted in a back ward direction is called the Front-to-Back ratio (FBR) of the antenna. It is also defined as the gain in a specified direction to the gain in a direction 180 deg. And it is usually expressed in dB (decibels).

Antenna Theorems: Four theorems can be applicable to antenna and field theory. They are 1. Reciprocity Theorem.

2. Equality of directional patterns.

3. Equality of effective lengths.

4. Equality of impedances.

Reciprocity Theorem:

Statement: If an e.m.f. or voltage is applied to the terminals of an antenna1 and the current measured at the terminals of another antenna 2, then same amount of current will be obtained at the terminals of antenna 1 if same e.m.f. or voltage is applied to the terminals of antenna 2.

Or

If a current I1 at the terminals of antenna 1 includes an e.m.f. E_{21} at the open terminals of antenna 2 and a current I_2 at the terminals of antenna 2 induces an e.m.f. E_{12} at the open terminals of antenna 1then $E_{12}=E_{21}$ provided $I_1=I_2$.

Proof: To prove the reciprocity theorem assume the medium between two antennas is linear, passive and isotropic so that it can be replaced by a four terminal network as shown in below figures.

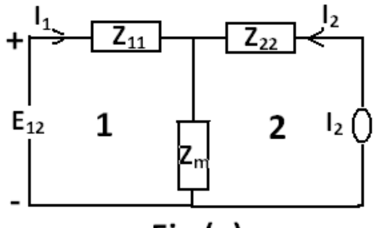


Fig (a)

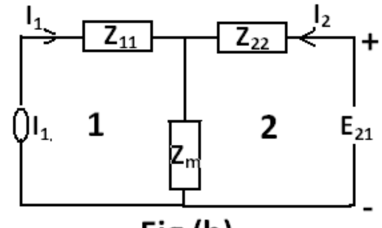


Fig (b)

Where Z11, Z22 are the self impedances of antennas 1 and 2., Zm is mutual impedance between two antennas. Applying KVL to figure (a) of mesh 2 , we get

$$(Z_{22}+Z_m)\ I_2 – Z_m I_1 = 0$$

$$I_2 = Z_m I_1 / (Z_{22}+Z_m) \text{ ……………………..(1)}$$

From mesh 1 we get

$$(Z_{11}+Z_m)\ I_1 – Z_m I_2 = E_{12}$$

$$(Z_{11}+Z_m)\ I_1 – Z_m^2 I_1/(Z_{22}+Z_m) = E_{12}$$

$$I_1[(Z_{11}+Z_m)\ (Z_{22}+Z_m) – Z_m^2] / (Z_{22}+Z_m) = E_{12}$$

$$I_1[Z_{11}\ Z_{22}+ Z_{11}Z_m + Z_{22}Z_m+ Z_m^2 – Z_m^2] / (Z_{22}+Z_m) = E_{12}$$

$$I_1 = E_{12}(Z_{22}+Z_m) /[Z_{11}\ Z_{22} + Z_m\ (Z_{11}+Z_{22})] \text{……………………(2)}$$

Substituting equation (2) into (1), we get

$$I_2 = Z_m\ E_{12}(Z_{22}+Z_m) /[Z_{11}\ Z_{22} + Z_m\ (Z_{11}+Z_{22})(\ Z_{22}+Z_m)$$

$$I_2 = Z_m\ E_{12}/[Z_{11}\ Z_{22} + Z_m\ (Z_{11}+Z_{22})]$$

Similarly the current I1 can be obtained from figure (b). By symmetry suffix 2 may be replaced by 1 vice-versa, we get

$$I_1 = Z_m\ E_{21}/[Z_{22}\ Z_{11} + Z_m\ (Z_{22}+Z_{11})] \text{…….(3)}$$

According to reciprocity theorem I1=I2. i.e. equating equations 2 and 3 we get

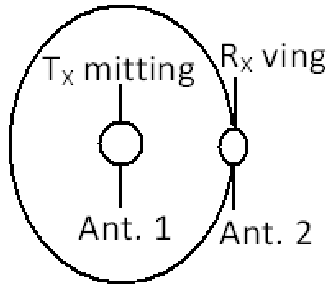
$$Z_m\ E_{12}/[Z_{11}\ Z_{22} + Z_m\ (Z_{11}+Z_{22})] = Z_m\ E_{21}/[Z_{22}\ Z_{11} + Z_m\ (Z_{22}+Z_{11})]$$

Therefore $\boxed{E_{12}= E_{21}}$ Hence proved.

Equality of directional patterns:

Statement: The directional patterns of transmitting and receiving antennas are same if all the media are linear, passive, isotropic and reciprocity theorem holds good.

Proof: Consider two antennas 1 and 2. Let antenna 1 as transmitting antenna (Test antenna) and antenna 2 as receiving antenna (Subject antenna). Keeping the antenna 1 at the center of the observation circle, the receiving antenna 2 is moved along the surface of the observation circle as shown in the figure.



Now if a voltage E is applied at transmitting antenna and the resulting current I at the terminals of receiving antenna is measured. If the process is reversed i.e. the same voltage E is applied to antenna 2 and resulting current I is measured at antenna 1. According to reciprocity theorem, for every position of antennas the ratio E/I is same. Thus it is proved radiation pattern or directional patterns of the antennas are same.

Equality of effective Lengths:

For transmitting antenna: Effective length of a transmitting antenna is defined as the length of a linear antenna that has a current I(c) at all points along its length and that radiates the electric field as shown in below figure.

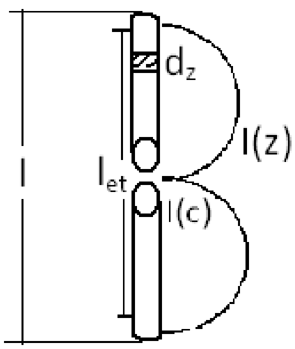


Fig (a)

From the figure,

$$I(c)l_{et} = \int I(z)\, d_z$$

$$\text{Or } l_{et} = [1/I(c)] \int I(z)\, d_z$$

Where l_{et} is effective length of transmitting antenna and

l is Actual or physical length of antenna.

If an e.m.f. or voltage E is applied at the center point then

$I(c) = \mathbf{E}/Z_t$ where Z_t is transmitting antenna impedance.

Applying reciprocity theorem then $\mathbf{E}=\mathbf{E}_{12}$ and $I(z)=I_2$.

For receiving antenna: The effective length of receiving antenna may be defined as the ratio of open circuit voltage developed at the terminals of antenna and received field strength i.e. $l_{er} = v/\mathbf{E}$. Now consider the same transmitting antenna for receiving case also but with center terminals shorted as shown in figure.

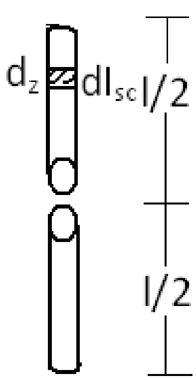


Fig (b)

Let an electromagnetic field strength Ezi is incident on the antenna which causes a voltage Ezi dz to induce in the element dz. Now applying reciprocity theorem we can write

$E_{zi}d_z = E_{21}$ and $dI_{sc} = I_1$

i.e. $(E_{12}/I_2) = (E_{21}/I_1)$ or $[E/I(z)] = (E_{zi}\, d_z)/dI_{sc}$

or $dI_{sc} = E_{zi}\, d_z\, I(z)/E$

$$\int dI_{sc} = \int E_{zi} d_z \, I(z)/E$$

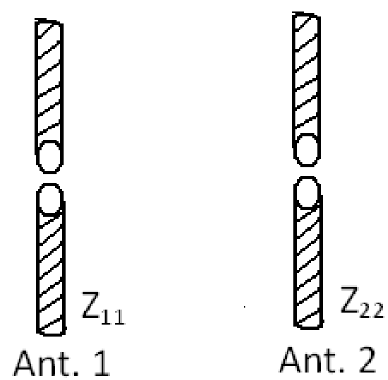
for the entire length of antenna $E_{zi} = E_z$

Therefore $I_{er} = I_{et}$

Equality of Impedances:

Statement: If an antenna is used either transmitting or receiving antenna its impedance will be the same.

Proof: We can prove this physically. Since the antenna is the same circuit irrespective of mode of excitation and hence the impedance of an antenna remains same whether transmitting or receiving. Let there are two antennas with a wide separation in between. If antenna 2 is quite away from antenna 1so that the mutual impedance between the two is neglected. Thus the self impedance of antenna 1 is given by $E_1=Z_{11}I_1+Z_{12}\,I_2 = Z_{11}I_1+(0)I_2 = E_1/I_1$ = self impedance of antenna 1. But this assumption not true when antenna 1 is receiving because during reception it is the mutual impedance (Z_{12}) between the two antennas which provides coupling. If an equivalent antenna 1 is drawn under load (Z_L) then $Z_{12}I_2$ acts as a voltage generator as shown in below figure.



Under open circuit condition $E= Z_{11}I_1+Z_{12}I_2$

$$E_{OC}=Z_{11}(0)+Z_{12}I_2 \text{(Since } I_1=0)$$

$$E_{OC}=Z_{12}I_2$$

Under short circuit condition $E=Z_{11}I_1+Z_{12}I_2$

$$0=Z_{11}I_{SC}+Z_{12}I_2 \quad \text{(Since } E=0 \text{ and } I_1=I_{SC})$$

$$I_{SC}= - Z_{12}I_2/Z_{11}.$$

Radiation: It is possible in theory to obtain expressions for the electric and magnetic field strengths **E** and **H** directly in terms of charge and current densities ρ and **J.** It is possible if we first set up potentials in terms of the charge and current densities and then to obtain the electric and magnetic fields from these potentials. There are 3 methods of determining the suitable potentials for the electromagnetic field. They are

1. Heuristic Approach.
2. Maxwell's equations Approach.
3. Solving of differential equations.

1. Heuristic Approach: In this method we will use the potentials already developed for static fields and we modify them suitable for time-varying fields as follows: In electrostatics a scalar potential v was set up related to the electric charge distribution as

$$V(r) = \frac{1}{4\pi\epsilon}\int \frac{\rho(r')}{R}\,dv'$$

From this the electric field was obtained through $\mathbf{E} = -\nabla V$

Similarly in steady magnetic fields the vector potential A was set up and related to the current density as

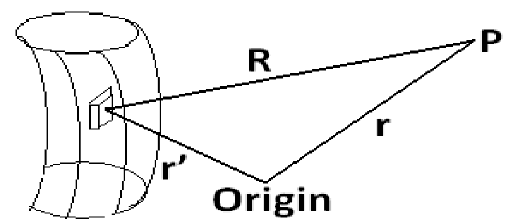
$$\mathbf{A}(r) = \frac{\mu}{4\pi}\int \frac{\mathbf{J}(r')}{R}\,dv'$$

From this magnetic potential the magnetic field was obtained through $\mu\mathbf{H} = \nabla \times \mathbf{A}$

The sources of the electromagnetic field are current and charge distributions that vary with time, so for time variations the potentials can be guessed as

$$\mathbf{A}(r,t) = \frac{\mu}{4\pi}\int \frac{\mathbf{J}(r',t)}{R}\,dv'$$

$$V(r,t) = \frac{1}{4\pi\epsilon}\int \frac{\rho(r',t)}{R}\,dv'$$



Where $R = r - r'$

But according to time varying theory (wave theory) there must be some finite propagation time for electromagnetic waves. The propagation delay for time varying fields is R/v seconds. Including this propagation delay the potentials are given as

$$\mathbf{A}(r,t-R/v) = \frac{\mu}{4\pi}\int \frac{\mathbf{J}(r',t-R/v)}{R}\,dv'$$

$$V(r,t-R/v) = \frac{1}{4\pi\epsilon}\int \frac{\rho(r',t-R/v)}{R}\,dv'$$

The above potentials have been delayed or retarded by an amount R/v so these potentials can also be called as Retarded or Delayed potentials. From these potentials we can find **H** and **E** using the equations

$$\mu\mathbf{H} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla V$$

2. Maxwell's equations Approach: In this method we start with Maxwell's equations and from these we derive differential equations that the potentials must satisfy as follows

Maxwell's equations in time-varying differential form are given by

$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t}$ +**J**………………… **(1)**

$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t}$ ………………………. **(2)**

$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon}$ …………………………... **(3)**

$\nabla \cdot \mathbf{H} = 0$ ……………………………. **(4)**

Where **J** and **ρ** are the current and charge densities respectively and they are related by equation of continuity

$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$ ……………………. **(5)**

Equation 4 is satisfied if H is represented as the curl of some vector. This leads to

$\mu \mathbf{H} = \nabla \times \mathbf{A}$………………………... **(6)**

Substitution of equation 6 into 2 gives

$\nabla \times (\mathbf{E} + \frac{\partial \mathbf{A}}{\partial \mathbf{t}}) = 0$ ………………. **(7)**

Equation 7 is satisfied if the inside the brackets is represented as the gradient of a scalar. i.e.

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V$$

$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t}$ ……………………. **(8)**

Substitution of 6 and 8 into 1 gives

$$\nabla \times \frac{(\nabla \times \mathbf{A})}{\mu} = \varepsilon \left(-\nabla V - \frac{\partial^2 \mathbf{A}}{\partial t^2} \right) + \mathbf{J}$$

The vector identity $\nabla \times (\nabla \times \mathbf{A}) = -\nabla^2 \mathbf{A} + \nabla(\nabla \cdot \mathbf{A})$

Can be used here. By doing so we get.

$$\nabla(\nabla\cdot\bar{A}) - \nabla^2\bar{A} = \mu\bar{J} - \mu\varepsilon\nabla\frac{\partial V}{\partial t} - \mu\varepsilon\frac{\partial^2\bar{A}}{\partial t^2}$$

$$\therefore \quad \boxed{\nabla^2\bar{A} - \nabla(\nabla\cdot\bar{A}) = -\mu\bar{J} + \mu\varepsilon\nabla\frac{\partial V}{\partial t} + \mu\varepsilon\frac{\partial^2\bar{A}}{\partial t^2}}$$

……………….. **(9)**

It is one of the required differential equation. The second one may be obtained by substituting 8 into 3

$$\nabla\cdot\left(-\nabla V - \frac{\partial\bar{A}}{\partial t}\right) = \frac{\rho_v}{\varepsilon}$$

$$\therefore \quad \boxed{\nabla^2 V + \nabla\cdot\frac{\partial\bar{A}}{\partial t} = -\frac{\rho_v}{\varepsilon}}$$

………………………………………. **(10)**

Equations 9 and 10 do not yield a unique solution for the potentials. Why because both equations contain the unknown **A** and v. So these equations are called as coupled equations. The reason for not getting the unique solution can be explained with the help of "Helmholtz Theorem". i.e. any vector field due to a finite source is specified uniquely if both the curl and divergence of the field are specified.

Examining the equations 9 and 10 shows that if the divergence of vector A is set equal to , $-\mu\epsilon\dot{v}$ these equations become uncoupled and reduces to standard wave equations as

$$\nabla^2\bar{A} - \nabla\left(-\mu\varepsilon\frac{\partial V}{\partial t}\right) = -\mu\bar{J} + \mu\varepsilon\nabla\frac{\partial V}{\partial t} + \mu\varepsilon\frac{\partial^2\bar{A}}{\partial t^2}$$

$$\therefore \quad \nabla^2\bar{A} + \mu\varepsilon\nabla\frac{\partial V}{\partial t} = -\mu\bar{J} + \mu\varepsilon\nabla\frac{\partial V}{\partial t} + \mu\varepsilon\frac{\partial^2\bar{A}}{\partial t^2}$$

$$\therefore \quad \boxed{\nabla^2\bar{A} - \mu\varepsilon\frac{\partial^2\bar{A}}{\partial t^2} = -\mu\bar{J}}$$

Similarly

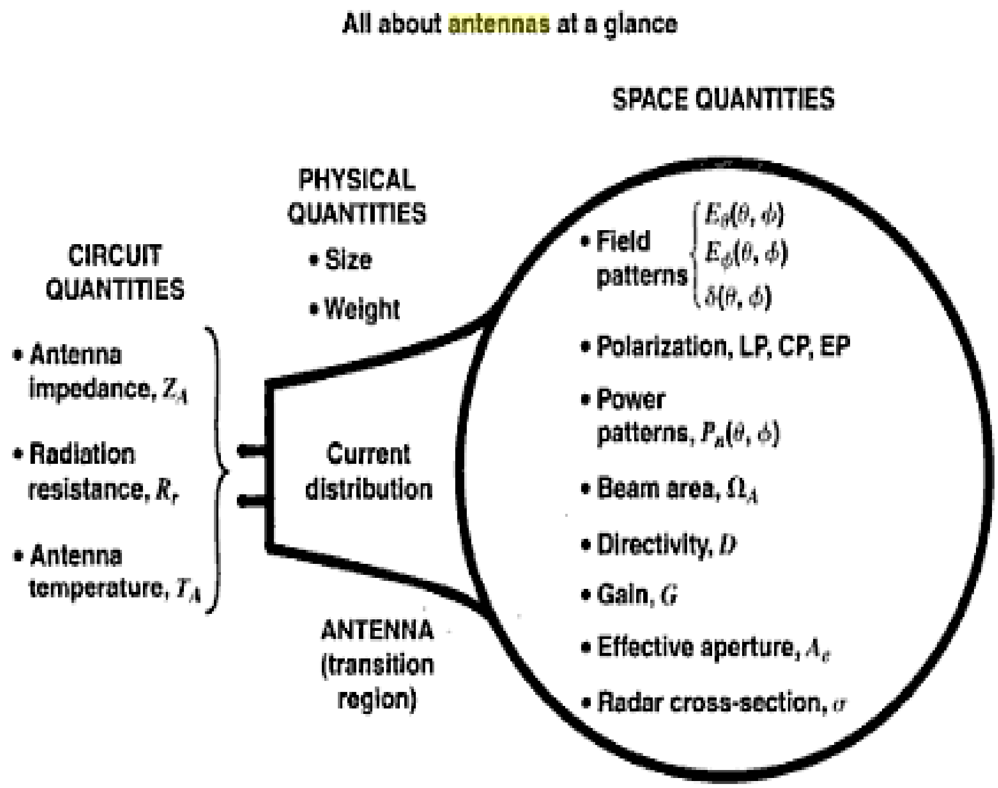
$$\nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot\bar{A}) = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 V + \frac{\partial}{\partial t}\left(-\mu\varepsilon\frac{\partial V}{\partial t}\right) = -\frac{\rho_v}{\varepsilon}$$

$$\therefore \quad \boxed{\nabla^2 V - \mu\varepsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\varepsilon}}$$

Hence the choice used $\nabla\cdot A = -\mu\epsilon\dot{v}$ is called the "Lorentz gauge Condition".

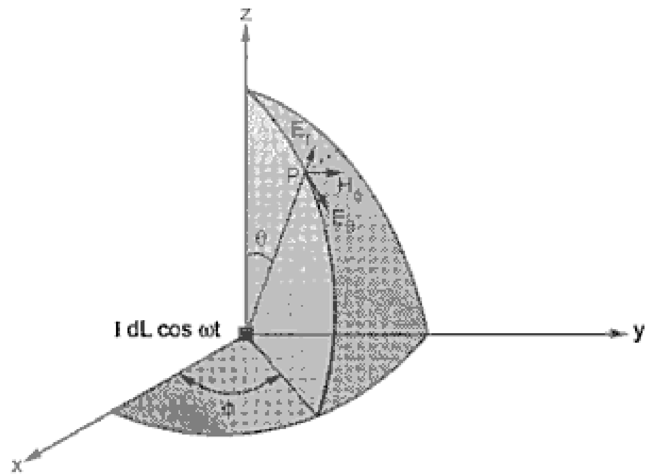
All about antennas: Different parameters studied under antenna basics can be divided into 3 regions of quantities as shown in the below figure.



UNIT-2 Thin Linear Wire Antennas

Alternating Current Element or Oscillating electric dipole: A current element **Idl** refers to a filamentary current **I** flowing along an elemental length **dl.** This is approximated when a current **I** flows in a very short length of thin wire, If the length **dl** considered is so short that the current is essentially constant along the length. Although an isolated current element may appear to be a very unreal concept, any physical circuit or Antenna carrying current may be considered to consists of a large number of such elements joined end-to-end. Therefore the current element is a basic building block for any type of practically existing antennas.

The below figure shows an alternating current element **Idl cosωt** located at the origin of a spherical coordinate system. The problem is to calculate the electromagnetic field at an arbitrary point P.



Electromagnetic field at point P when an alternating current element I dL cos ω t placed at origin

To solve this problem, the first step is to obtain the vector potential **A** at p. Assuming that the wave travelling at the distant point in the positive z-direction. Hence from Heuristic approach we can write

$$A_z = \frac{\mu}{4\pi} \int_v \frac{\bar{J}\left(t - \frac{r}{v}\right)}{R} \, dv'$$

The component of vector potential $\mathbf{A_Z}$ can be obtained by integrating the current density **J** over the volume. This includes integration over the cross section area of the current element and integration along its length. Buy the integration of current density **J** over a cross section area yields current **I.** Now this current is assumed to be constant along the length **dl.** Thus

$$A_z = \frac{\mu}{4\pi} \frac{I\, dL \cos\omega\left(t - \frac{r}{v}\right)}{r}$$

Here R=r because the current element is placed at the origin itself. From this the magnetic field strength **H** can be obtained using the relation $\mu \bar{H} = \nabla \times \bar{A}$

The curl of **A** in spherical coordinates gives the component of **H** in terms of $\mathbf{A_r}$**,** $\mathbf{A_\theta}$ **and** $\mathbf{A_\varphi}$.

From the above figure we get

$$A_r = A_z \cos\theta$$

$$A_\theta = -A_z \sin\theta$$

$$A_\phi = 0$$

Hence $\bar{A}$ is given by,

$$\nabla \times \bar{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}A_r - \frac{\partial}{\partial r}(r A_\phi)\right]\bar{a}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial}{\partial\theta}(A_r)\right]\bar{a}_\phi$$

Now note that $\mathbf{A_\varphi}$.**=0** and because of symmetry $\frac{\partial}{\partial\varphi}$ =0.as no variation along φ direction. Thus first two terms in above equation can be neglected being zero.

$$\therefore \nabla \times \bar{A} = \frac{1}{r}\left[\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\bar{a}_\phi$$

Putting value of A_θ=-A_z Sinθ and A_r= A_z Sinθ. We get

$$\nabla \times \bar{A} = \frac{1}{r}\left[\frac{\partial}{\partial r}\{r(-A_z \sin\theta)\} - \frac{\partial}{\partial\theta}\{A_z \cos\theta\}\right]\bar{a}_\phi$$

Substituting value of A_z,

$$\nabla \times \bar{A} = \frac{\mu}{r}\left[\frac{\partial}{\partial r}\left\{(-r)\sin\theta \frac{I\, dL \cos\omega\left(t - \frac{r}{v}\right)}{4\pi r}\right\} - \right.$$

$$\frac{\partial}{\partial\theta}\left\{\cos\theta\,\frac{I\,dL\cos\omega\left(t-\frac{r}{v}\right)}{4\pi r}\right\}\Bigg]\Bigg]\bar{a}_\phi$$

$$\nabla\times\bar{A} = \frac{\mu I\,dL}{4\pi r}\left\{\left[(-\sin\theta)\left\{\frac{-\omega\sin\omega\left(t-\frac{r}{v}\right)}{-v}\right\}\right]-\right.$$

$$\left.\left[\frac{(-\sin\theta)}{r}\left\{\cos\omega\left(t-\frac{r}{v}\right)\right\}\right]\right\}\bar{a}_\phi$$

$$\nabla\times\bar{A} = \frac{\mu I\,dL\sin\theta}{4\pi r}\left[\frac{-\omega\sin\omega\left(t-\frac{r}{v}\right)}{v}+\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r}\right]\bar{a}_\phi$$

$$\nabla\times\bar{A} = \frac{\mu I\,dL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega\left(t-\frac{r}{v}\right)}{rv}+\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r^2}\right]\bar{a}_\phi$$

Hence the magnetic field **H** is given by

$$\bar{H} = \frac{1}{\mu}[\nabla\times\bar{A}]$$

Putting value of $\nabla\times\bar{A}$ from equation (9), we get,

$$\bar{H} = \frac{I\,dL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega\left(t-\frac{r}{v}\right)}{rv}+\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r^2}\right]\bar{a}_\phi$$

Equation (10) indicates that the magnetic field $\bar{H}$ exists only in ϕ direction.

$$\therefore \quad H_\phi = \frac{I\,dL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega\left(t-\frac{r}{v}\right)}{rv}+\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r^2}\right]$$

Let $\left(t-\frac{r}{v}\right)=t'$

$$H_\phi = \frac{I\,dL \sin\theta}{4\pi}\left[\frac{-\omega \sin\omega t'}{r v} + \frac{\cos\omega t'}{r^2}\right]$$

After calculating the magnetic field, now let us calculate the electric field given by,

$$\nabla \times \overline{H} = \varepsilon \frac{\partial \overline{E}}{\partial t}$$

$$\therefore \qquad \partial \overline{E} = \frac{1}{\varepsilon}(\nabla \times \overline{H})\, dt \qquad \text{... separating variables}$$

Integrating with respect to corresponding variables, we get,

$$\overline{E} = \frac{1}{\varepsilon}\int \nabla \times \overline{H}\; dt$$

Let us calculate each term of $\nabla \times \overline{H}$ separately.

From the definition of curl of a vector, the component in $\overline{a}_r$ direction is given by

$$(\nabla \times \overline{H})_r = \frac{1}{r\sin\theta}\left[\frac{\partial H_\phi \sin\theta}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right]$$

But $\qquad \dfrac{\partial}{\partial\phi} = 0$

$$\therefore \qquad (\nabla \times \overline{H})_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\{H_\phi \sin\theta\}\right]$$

Substituting value of H_ϕ

$$(\nabla \times \overline{H})_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left\{\frac{I\,dL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{r v} + \frac{\cos\omega t'}{r^2}\right]\sin\theta\right\}\right]$$

$$\therefore \qquad (\nabla \times \overline{H})_r = \frac{1}{r\sin\theta}\cdot\frac{I\,dL}{4\pi}\left[\frac{-\omega\sin\omega t'}{r v} + \frac{\cos\omega t'}{r^2}\right]\left\{\frac{\partial}{\partial\theta}\sin^2\theta\right\}$$

$$\therefore \qquad (\nabla \times \overline{H})_r = \frac{I\,dL}{(r\sin\theta)\,4\pi}\left[\frac{-\omega\sin\omega t'}{r v} + \frac{\cos\omega t'}{r^2}\right](2\sin\theta\cos\theta)$$

$$\therefore \qquad (\nabla \times \overline{H})_r = \frac{2\,I\,dL\cos\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{v r^2} + \frac{\cos\omega t'}{r^3}\right]$$

Let us calculate the component in $\overline{a}_\theta$ direction

$$(\nabla \times \overline{H})_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(r H_\phi)}{\partial r}\right]$$

But again $\qquad \dfrac{\partial}{\partial\phi} = 0$

$$\therefore \qquad (\nabla \times \overline{H})_\theta = \frac{1}{r}\left[-\frac{\partial}{\partial r}\{r H_\phi\}\right]$$

Substituting value of H_ϕ

$$(\nabla \times \overline{H})_\theta = \frac{-I\,dL\sin\theta}{4\pi r}\frac{\partial}{\partial r}\left\{\frac{-r\omega\sin\omega\left(t-\frac{r}{v}\right)}{r v} + \frac{r\cos\omega\left(t-\frac{r}{v}\right)}{r^2}\right\}$$

$$\therefore \qquad (\nabla \times \overline{H})_\theta = \frac{-I\,dL\sin\theta}{4\pi r}\frac{\partial}{\partial r}\left\{\frac{-\omega\sin\omega\left(t-\frac{r}{v}\right)}{v} + \frac{\cos\omega\left(t-\frac{r}{v}\right)}{r}\right\}$$

$$\therefore \quad (\nabla\times\overline{H})_\theta = -\frac{I\,dL\sin\theta}{4\pi\, r}\left\{\left[\frac{-\omega\cos\omega\left(t-\frac{r}{v}\right)}{v}\right]\left[-\frac{\omega}{v}\right]+\right.$$

$$\left.\frac{1}{r^2}\left[(r)\sin\omega\left(t-\frac{r}{v}\right)\left(\frac{\omega}{v}\right)-\cos\omega\left(t-\frac{r}{v}\right)\right]\right\}$$

$$\therefore \quad (\nabla\times\overline{H})_\theta = -\frac{I\,dL\sin\theta}{4\pi}\left[\frac{\omega^2\cos\omega\left(t-\frac{r}{v}\right)}{v^2 r}+\frac{\omega\sin\omega\left(t-\frac{r}{v}\right)}{v\, r^2}-\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r^3}\right]$$

Finally the component of $(\nabla\times\overline{H})$ in $\bar{a}_\phi$ direction is zero.

From equation (13), the component of $\overline{E}$ in $\bar{a}_r$ direction is given by

$$E_r = \frac{1}{\varepsilon}\int (\nabla\times\overline{H})_r\, dt$$

Putting value of $(\nabla\times\overline{H})_r$ from equation (14),

$$E_r = \frac{1}{\varepsilon}\int \frac{2\,I\,dL\cos\theta}{4\pi}\left[\frac{-\omega\sin\omega\, t'}{v\, r^2}+\frac{\cos\omega\, t'}{r^3}\right]dt$$

$$= \frac{1}{\varepsilon}\int \frac{2\,I\,dL\cos\theta}{4\pi}\left[\frac{-\omega\sin\omega\left(t-\frac{r}{v}\right)}{v\, r^2}+\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r^3}\right]dt$$

$$= \frac{2\,I\,dL\cos\theta}{4\pi\varepsilon}\left[\frac{\omega\cos\omega\left(t-\frac{r}{v}\right)}{v\, r^2}\left(\frac{1}{\omega}\right)+\frac{\sin\omega\left(t-\frac{r}{v}\right)}{r^3}\left(\frac{1}{\omega}\right)\right]$$

$$= \frac{2\,I\,dL\cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega\left(t-\frac{r}{v}\right)}{v\, r^2}+\frac{\sin\omega\left(t-\frac{r}{v}\right)}{\omega\, r^3}\right]$$

Put $\left(t-\frac{r}{v}\right)=t'$

$$\therefore \quad E_r = \frac{2\,I\,dL\cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega\, t'}{v r^2}+\frac{\sin\omega\, t'}{\omega r^3}\right]$$

Similarly the component of $\overline{E}$ in $\bar{a}_\theta$ direction is given by,

$$E_\theta = \frac{1}{\varepsilon}\int (\nabla\times\overline{H})_\theta\, dt$$

Substituting value of $(\nabla\times\overline{H})_\theta$

$$E_\theta = \frac{1}{\varepsilon}\int \frac{-I\,dL\sin\theta}{4\pi\, r}\left[\frac{\omega^2\cos\omega\left(t-\frac{r}{v}\right)}{v^2 r}+\frac{\omega\sin\omega\left(t-\frac{r}{v}\right)}{v r^2}-\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r^3}\right]dt$$

$$\therefore E_\theta = \frac{-I\,dL\sin\theta}{4\pi\varepsilon}\int\left[\frac{\omega^2\cos\omega\left(t-\frac{r}{v}\right)}{v^2 r}+\frac{\omega\sin\omega\left(t-\frac{r}{v}\right)}{vr^2}-\frac{\cos\omega\left(t-\frac{r}{v}\right)}{r^3}\right]dt$$

$$\therefore E_\theta = \frac{-I\,dL\sin\theta}{4\pi\varepsilon}\left[\frac{\omega^2\sin\omega\left(t-\frac{r}{v}\right)}{v^2 r}\left(\frac{1}{\omega}\right)+\frac{-\omega\cos\omega\left(t-\frac{r}{v}\right)}{vr^2}\left(\frac{1}{\omega}\right)-\frac{\sin\omega\left(t-\frac{r}{v}\right)}{r^3}\left(\frac{1}{\omega}\right)\right]$$

$$\therefore E_\theta = \frac{-I\,dL\sin\theta}{4\pi\varepsilon}\left[\frac{\omega\sin\omega\left(t-\frac{r}{v}\right)}{v^2 r}-\frac{\cos\omega\left(t-\frac{r}{v}\right)}{vr^2}-\frac{\sin\omega\left(t-\frac{r}{v}\right)}{\omega r^2}\right]$$

$$\therefore E_\theta = \frac{I\,dL\sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t'}{v^2 r}+\frac{\cos\omega t'}{vr^2}+\frac{\sin\omega t'}{\omega r^3}\right]$$

Significance of Field Components: Consider the expression for the component Hφ given by

$$H_\phi = \frac{I\,dL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rv}+\frac{\cos\omega t'}{r^2}\right]$$

We can note the following observations.

1. The first term varies inversely with distance r. This term is called radiation or distant field.
2. The second term varies inversely with square of distance, r. This term is called the induction field. When r is small, the points are very close to the current element and the induction field term is dominating. But when the points are far away from the current element, then induction field term is negligible as compared to the radiation field.
3. The condition at which the amplitudes of both radiation and induction fields are equal is given by

$$\frac{1}{r^2} = \frac{\omega}{rv}$$

$$\therefore \quad r = \frac{v}{\omega} = \frac{v}{2\pi f} = \frac{(v/f)}{2\pi} = \frac{\lambda}{2\pi} = \frac{\lambda}{6}$$

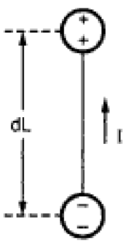
4. The induction field term is similar to the expression for the magnetic field strength due to the current element derived from Biot-Savart's law, extended for alternating current element.
5. For steady currents the radiation field term is absent.
6. The radiation field term indicates flow of energy away from the current element. While the induction field term indicates the energy stored in the field during one-quarter of the cycle which is returned back during next cycle.

Now consider the expressions of the components $\mathbf{E_r}$ and $\mathbf{E_\theta}$ we can note the following observations.

1. The component $\mathbf{E_\theta}$ has both the induction and radiation terms along with a term which varies inversely with the cube of the distance, r.

2. The component $\mathbf{E_r}$ has only induction term along with a term which varies inversely with the cube of the distance, r.
3. In both the field component expressions the term which varies inversely with the cube of the distance, r is called electrostatic field or simply electric field.

Relation between a current element and electric dipole: The alternating current element and electric dipole can be approximately equated using the equation $\frac{dq}{dt} = I \cos \omega t$



In order to obtain a physical approximation of an isolated current element, we can terminate the current element in two small spheres or disks on which the charge could accumulate. If the wire is very thin compared with the radius of the spheres, the current in the wire will be uniform. In addition, the radii of the spheres should be small compared with dl and in turn dl should be very much shorter than a wavelength. By satisfying these conditions the alternating current element can be used as an electric dipole.

Power radiated by current element: The power flow per unit area at any point,p will be given by the pointing vector at that point. For the current element placed along the positive z-direction and at the origin of the spherical coordinates can have P_θ,P_r and P_φ . The components of pointing vector are given by

$$P_r = E_\theta H_\phi$$
$$P_\theta = -E_r H_\phi$$
$$P_\phi = E_\phi H_r$$

Out of these three, P_φ will be zero due to symmetry. The θ component of power from the pointing vector can be found as where $P_\theta = -E_r H_\phi$

$$E_r = \frac{2 I dL \cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t'}{cr^2} + \frac{\sin\omega t'}{\omega r^3}\right]$$

$$\text{and} \quad H_\phi = \frac{I dL \sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rc} + \frac{\cos\omega t'}{r^2}\right]$$

The θ component of the instantaneous Poynting vector is given by

$$P_\theta = -E_r H_\phi$$

$$= \frac{-2 I dL \cos\theta}{4\pi\varepsilon}\left[\frac{\cos\omega t'}{cr^2} + \frac{\sin\omega t'}{\omega r^3}\right]\frac{I dL \sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rc} + \frac{\cos\omega t'}{r^2}\right]$$

$$= \left[\frac{2 I^2 dL^2 \sin\theta\cos\theta}{16\pi^2\varepsilon}\right]\left[\frac{-\omega\sin\omega t'\cos\omega t'}{c^2 r^3} + \frac{\cos^2\omega t'}{cr^4} - \frac{\omega\sin^2\omega t'}{\omega cr^4} + \frac{\sin\omega t'\cos\omega t'}{\omega r^5}\right]$$

Using property $2\sin\theta\cos\theta = \sin 2\theta$,

$$= \frac{I^2 dL^2 \sin 2\theta}{16\pi^2\varepsilon}\left[\frac{\omega\sin 2\omega t'}{2c^2 r^3} - \frac{\cos^2\omega t'}{cr^4} + \frac{\omega\sin^2\omega t'}{\omega cr^4} - \frac{\sin 2\omega t'}{2\omega r^5}\right]$$

$$= \left[\frac{I^2 dL^2 \sin 2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega\sin 2\omega t'}{2c^2 r^3} + \frac{1}{c r^4}(\sin^2\omega t' - \cos^2\omega t') - \frac{\sin 2\omega t'}{2\omega r^5}\right]$$

Consider the middle term inside the second square bracket,

$$\frac{1}{cr^4}[\sin^2 \omega t' - \cos^2 \omega t'] = \frac{1}{c\,r^4}\left[\frac{1-\cos 2\omega t'}{2} - \left(\frac{1+\cos 2\omega t'}{2}\right)\right]$$

$$= \frac{1}{c\,r^4}\left[\frac{-2\cos 2\omega t'}{2}\right]$$

$$= \frac{-\cos 2\omega t'}{cr^4}$$

Putting value of the term considered back in the original expression,

$$P_\theta = \left[\frac{I^2\ dL^2\ \sin 2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega \sin 2\omega t'}{2c^2 r^3} - \frac{\cos 2\omega t'}{cr^4} - \frac{\sin 2\omega t'}{2\omega r^5}\right]$$

The average value of sin2ωt' or cos ωt' over a complete cycle is zero. Therfore for any value of r, the average of P_θ over complete cycle is zero. P_θ represents only back and forth of power in θ direction without any net or average flow. The radial pointing vector is given by

$$P_r = E_\theta H_\phi$$

$$= \frac{I\,dL\sin\theta}{4\pi\varepsilon}\left[\frac{-\omega\sin\omega t'}{c^2 r} + \frac{\cos\omega t'}{cr^2} + \frac{\sin\omega t'}{\omega r^3}\right]\frac{I\,dL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rc} + \frac{\cos\omega t'}{r^2}\right]$$

$$= \left[\frac{I^2 dL^2 \sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega^2\sin^2\omega t'}{c^3 r^2} - \frac{\omega\sin\omega t'\cos\omega t'}{c^2 r^3} - \frac{\omega\sin\omega t'\cos\omega t'}{c^2 r^3}\right.$$

$$\left. + \frac{\cos^2\omega t'}{cr^4} - \frac{\omega\sin^2\omega t'}{\omega c r^4} + \frac{\sin\omega t'\cos\omega t'}{\omega r^5}\right]$$

$$= \left[\frac{I^2 dL^2 \sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega^2\sin^2\omega t'}{c^3 r^2} - \frac{\omega\sin 2\omega t'}{2c^2 r^3} - \frac{\omega\sin 2\omega t'}{2\,c^2 r^3}\right.$$

$$\left. + \frac{\cos^2\omega t'}{cr^4} - \frac{\sin^2\omega t'}{\omega c r^4} + \frac{\sin 2\omega t'}{2\,\omega r^5}\right]$$

$$= \left[\frac{I^2 dL^2 \sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega^2}{c^3 r^2}\left(\frac{1-\cos 2\omega t'}{2}\right) - \frac{\omega\sin 2\omega t'}{2c^2 r^3} - \frac{\omega\sin 2\omega t'}{2c^2 r^3}\right.$$

$$\left. + \left(\frac{1+\cos 2\omega t'}{2\,cr^4}\right) - \left(\frac{1-\cos 2\omega t'}{2\,cr^4}\right) + \frac{\sin 2\omega t'}{2\,\omega r^5}\right]$$

$$= \left[\frac{I^2 dL^2 \sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega^2(1-\cos 2\omega t')}{2\,c^3\ r^2} - \frac{\omega\sin 2\omega t'}{c^2 r^3} + \frac{\cos 2\omega t'}{2\,cr^4} + \frac{\cos 2\omega t'}{2\,cr^4} + \frac{\sin 2\omega t'}{2\omega r^5}\right]$$

$$= \left[\frac{I^2 dL^2 \sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\omega^2(1-\cos 2\omega t')}{2c^3 r^2} - \frac{\omega\sin 2\omega t'}{c^2 r^3} + \frac{\cos 2\omega t'}{c\,r^4} + \frac{\sin 2\omega t'}{2\,\omega\,r^5}\right]$$

Rearranging the terms,

$$P_r = \left[\frac{I^2 dL^2 \sin^2\theta}{16\pi^2\varepsilon}\right]\left[\frac{\sin 2\omega t'}{2\,\omega\,r^5} + \frac{\cos 2\omega t'}{cr^4} - \frac{\omega\sin 2\omega t'}{c^2 r^3} + \frac{\omega^2(1-\cos 2\omega t')}{2\,c^3 r^2}\right]$$

The average value of radial pointing vector over a cycle will be due to part of the final term only and is

$$P_r = \left[\frac{I^2 dL^2 \sin^2\theta}{16\pi^2 \varepsilon}\right]\left[\frac{\omega^2}{2 r^2 c^3}\right]$$

$$P_r = \frac{\omega^2 I^2 dL^2 \sin^2\theta}{32\pi^2 r^2 c^3 \varepsilon}$$

$$\therefore \qquad P_r = \frac{1}{2\varepsilon c}\left(\frac{\omega I\, dL \sin\theta}{4\pi r c}\right)^2$$

But for free space intrinsic impedance $\eta_0 = \frac{1}{\varepsilon c}$.

$$\therefore \qquad \boxed{P_r = \frac{\eta_0}{2}\left(\frac{\omega I\, dL \sin\theta}{4\pi r c}\right)^2}$$

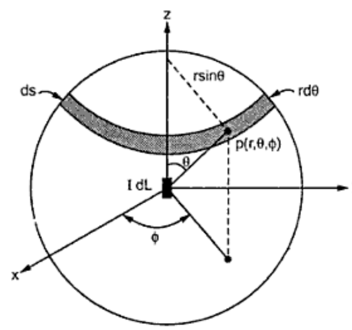
Now the total power radiated by the current element can be calculated by integrating the radial pointing vector over a spherical surface. The differential area on the spherical shell will be taken as the strip $da=2\pi r^2 \sin\theta\, d\theta$. Then the total power radiated is calculated by integrating average radial power over the spherical surface as

$$\text{Power} = \oint_{surface} P_r\, ds$$

$$= \oint_{surface}\left[\frac{\eta_0}{2}\left(\frac{\omega I\, dL \sin\theta}{4\pi r c}\right)^2 (2\pi r^2 \sin\theta\, d\theta)\right]$$

$$= \oint_{surface}\left(\frac{\eta_0}{2}\right)\left(\frac{\omega^2 I^2 dL^2 \sin^2\theta}{16\pi^2 r^2 c^2}\right)(2\pi r^2 \sin\theta)\, d\theta$$

$$= \oint_{surface} \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2} \sin^3\theta\, d\theta$$



$$= \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2} \oint_{surface} \sin^3\theta\, d\theta$$

In spherical co-ordinate system, θ varies from 0 to π . Hence putting limits of integration as,

$$\text{Power} = \frac{\eta_0 \omega^2 I^2 dL^2}{16\pi c^2}\int_0^{\pi} \sin^3\theta\, d\theta$$

$$\therefore \qquad \text{Power} = \frac{\eta_0 \omega^2 I^2 dL^2}{8\pi c^2}\int_0^{\pi} \sin^3\theta\, d\theta \;\ldots\ldots \int_0^{\pi} \sin^n\theta\, d\theta = 2\int_0^{\pi/2} \sin^n\theta\, d\theta$$

Using the reduction formula for calculating integral. By the reduction formula,

$$\int_0^{\pi/2} \sin^n x\, dx = \left[\frac{n-1}{n}\right]\left[\frac{\pi}{2}\right] \qquad \text{if n is even}$$

$$= \left[\frac{n-1}{n}\right] \qquad \text{if n is odd}$$

Hence n is 3 i.e. odd, hence we can write,

$$\int_0^{\pi/2} \sin^3\theta\, d\theta = \frac{3-1}{3} = \frac{2}{3}$$

Substituting this value in the expression of power, we get,

$$\text{Power} = \frac{\eta_0 \omega^2 I^2 dL^2}{8\pi c^2}\left(\frac{2}{3}\right)$$

The power represented in the above equation is in terms of maximum or peak current. We know that

$$I_{eff} = \frac{I_m}{\sqrt{2}}$$

$$I_m = \sqrt{2}\ I_{eff}$$

Thus the power can be expressed interms of the effective current as

$$\text{Power} = \frac{\eta_0\, \omega^2 (\sqrt{2}\ I_{eff})^2\, dL^2}{12\,\pi c^2}$$

$$\therefore \quad \boxed{\text{Power} = \frac{\eta_0\, \omega^2\, I_{eff}^2\, dL^2}{6\pi c^2}}$$

For free space, $\eta_0 = 120\pi$ and

$$\frac{\omega}{c} = \frac{2\pi}{\lambda} \quad \text{i.e.} \quad \frac{\omega^2}{c^2} = \frac{4\pi^2}{\lambda^2}$$

Substituting values of η_0 and $\frac{\omega^2}{c^2}$ in above equation we get,

$$\text{Power} = \frac{(120\pi)\left(\frac{4\pi^2}{\lambda^2}\right) I_{eff}^2\ dL^2}{6\pi}$$

$$\therefore \quad \text{Power} = \frac{80\pi^2\ I_{eff}^2\ dL^2}{\lambda^2}$$

$$\therefore \quad \boxed{\text{Power} = 80\pi^2 \left(\frac{dL}{\lambda}\right)^2 I_{eff}^2}$$

The coefficient of I^2_{eff} has the dimensions of resistance and is called the radiation resistance (R_r) of the current element. Therefore for the current element is

$$R_r = 80\pi^2 \left(\frac{dL}{\lambda}\right)^2$$

Radiation resistance of Short Dipoles and Short Monopoles: The radiation resistance obtained for the current element can be applied to dipoles of short length up to λ/4 and for monopoles up to λ/8 only.. Because up to these lengths the dipoles and monopoles can be equal to the current element. For longer lengths of dipoles i.e. greater thanλ/4 and longer lengths of monopoles i.e. greater than λ/8, the current element phenomenon will be not applied. Therefore the radiation resistance of practical short dipole is one-quarter that of the current element of the same length. That is

$$R_{rad}\ (\text{short diople}) = \frac{1}{4}\left[80\pi^2 \left(\frac{l}{\lambda}\right)^2\right]$$

$$= 20\pi^2 \left(\frac{L}{\lambda}\right)^2$$

$$\therefore \quad \boxed{R_{rad}\ (\text{short diople}) \approx 200\left(\frac{L}{\lambda}\right)^2}$$

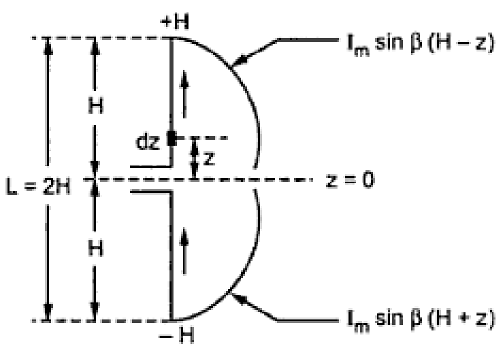
Similarly the radiation resistance of a practical short monopole is one-eighth that of current element of the same length or it is half of that of the dipole. That is

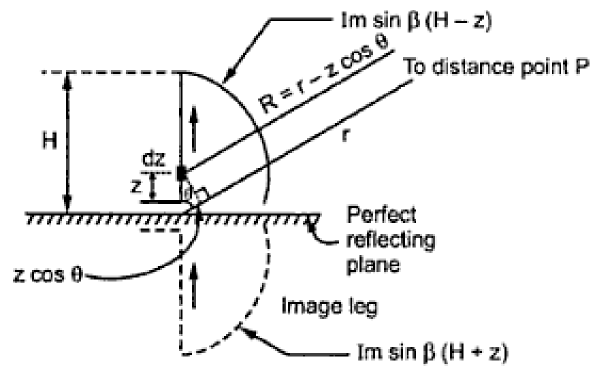
$$\therefore\ R_{rad}\ (\text{monopole}) = \frac{1}{2}\left[R_{rad}\ (\text{short dipole})\right]$$

$$= \frac{1}{2}\left[20\pi^2\left(\frac{l}{\lambda}\right)^2\right]$$

$$= 10\pi^2\left(\frac{l}{\lambda}\right)^2$$

Radiation from a quarter-wave Monopole or Half-wave dipole:





As shown in the above figure the distribution of current for a λ/2 dipole or λ/4 monopole can be assumed as sinusoidal. Then

$$I = I_m \sin\beta(H-z) \quad \text{for} \quad z > 0$$

$$I = I_m \sin\beta(H+z) \quad \text{for} \quad z < 0$$

Where Im is the maximum or peak current. The expression for the vector potential at a point p due to the current element Idz will be

$$dA_z = \frac{\mu I}{4\pi R} e^{-j\beta R}\, dz$$

Where R is the distance from the current element to the point p. The total vector potential at p due to all the current elements will be

$$A_z = \frac{\mu}{4\pi}\int_{-H}^{0}\frac{I}{R}e^{-j\beta R}\,dz + \frac{\mu}{4\pi}\int_{0}^{+H}\frac{I}{R}e^{-j\beta R}\,dz$$

For a large distance at P, R≈r in the denominator. But this approximation cannot be used in numerator because it is the phase factor which is important. Then the expression for $\mathbf{A_z}$ becomes

$$A_z = \frac{\mu I_m}{4\pi r}\left[\int_{-H}^{0}\sin\beta(H+z)\,e^{-j\beta r}\,e^{j\beta z\cos\theta}\,dz + \int_{0}^{+H}\sin\beta(H-z)\,e^{-j\beta r}\,e^{j\beta z\cos\theta}\,dz\right]$$

$$\therefore\ A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0}\sin\beta(H+z)\,e^{j\beta z\cos\theta}\,dz + \int_{0}^{+H}\sin\beta(H-z)\,e^{j\beta z\cos\theta}\,dz\right]$$

$$\therefore\ A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0} \sin\beta(H+z)\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{+H} \sin\beta(H-z)\, e^{j\beta z\cos\theta}\, dz\right]$$

For quarter wave monopole,

$$H = \frac{\lambda}{4} \text{ and}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\therefore \quad \sin\beta(H+z) = \sin(\beta H + \beta z) = \sin\left(\frac{\pi}{2} + \beta z\right)$$

$$\text{and} \quad \sin\beta(H-z) = \sin(\beta H - \beta z) = \sin\left(\frac{\pi}{2} - \beta z\right)$$

$$\text{But} \quad \sin\left(\frac{\pi}{2} + \beta z\right) = \sin\left(\frac{\pi}{2} - \beta z\right) = \cos\beta z$$

Substituting values of sine terms we get,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0} \cos\beta z\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{H} \cos\beta z\, e^{j\beta z\cos\theta}\, dz\right]$$

Now $\int_{-H}^{0} e^{+j\theta}\, d\theta = \int_{0}^{H} e^{-j\theta} d\theta$. Hence using this property, changing limits of integration of the first term.

$$\therefore \quad A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{H} \cos(-\beta z)\, e^{-j\beta z\cos\theta}\, dz + \int_{0}^{H} \cos\beta z\, e^{j\beta z\cos\theta}\, dz\right]$$

$$\therefore \quad A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{H} \cos\beta z\, e^{-j\beta z\cos\theta}\, dz + \int_{0}^{H} \cos\beta z\, e^{j\beta z\cos\theta}\, dz\right]$$

$$\dots \cos(-\theta) = \cos\theta$$

$$\therefore \quad A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{H} \cos\beta z\, (e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta})\, dz\right]$$

By Euler's identity,

$$e^{j\beta z\cos\theta} = \cos(\beta z\cos\theta) + j\sin(\beta z\cos\theta)$$

$$e^{-j\beta z\cos\theta} = \cos(\beta z\cos\theta) - j\sin(\beta z\cos\theta)$$

$$\therefore e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta} = 2\cos(\beta z\cos\theta)$$

Putting the value of the term inside the bracket we get,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_{0}^{H} (\cos\beta z)\, 2\,[\cos(\beta z\cos\theta)]\, dz$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \int_0^H 2\cos\beta z \cos(\beta z \cos\theta)\, dz$$

From the trigonometric identity,

$$2\cos A \cos B = \cos(A - B) + \cos(A + B)$$

Using this property we get,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \int_0^H \{\cos[\beta z + \beta z \cos\theta] + \cos[\beta z - \beta z \cos\theta]\}\, dz$$

$$\therefore \quad A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r} \int_0^{H=\lambda/4} \{\cos\beta z(1+\cos\theta) + \cos\beta z(1-\cos\theta)\}\, dz$$

Integrating with respect to z and putting value of $H = \lambda / 4$ as limit, we get,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin\beta z(1+\cos\theta)}{\beta(1+\cos\theta)} + \frac{\sin\beta z(1-\cos\theta)}{\beta(1-\cos\theta)}\right]_0^{\lambda/4}$$

Finding LCM,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{[\sin\beta z(1+\cos\theta)](1-\cos\theta) + [\sin\beta z(1-\cos\theta)](1+\cos\theta)}{\beta(1-\cos^2\theta)}\right]_0^{\lambda/4}$$

$$\therefore \quad A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\left[\sin\frac{\pi}{2}(1+\cos\theta)\right] + (1+\cos\theta)\left[\sin\frac{\pi}{2}(1-\cos\theta)\right]}{\sin^2\theta}\right]$$

$$\ldots \beta z = \frac{\pi}{2} \text{ and } (1-\cos^2\theta) = \sin^2\theta$$

Again using property,

$$\sin(\pi/2+\theta) = \sin(\pi/2-\theta) = \cos\theta$$

$$\therefore \quad \sin\left(\frac{\pi}{2}+\frac{\pi}{2}\cos\theta\right) = \sin\left(\frac{\pi}{2}-\frac{\pi}{2}\cos\theta\right) = \cos\left(\frac{\pi}{2}\cos\theta\right)$$

Substituting values of the sine terms we get,

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\cdot\cos\left(\frac{\pi}{2}\cos\theta\right) + (1+\cos\theta)\cdot\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]$$

$$\therefore \quad A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{2\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]$$

$$\therefore\ A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]$$

After finding the vector potential, the next step is to find the magnetic field using Maxwell's equation. The ϕ components of $\overline{H}$ is given by,

$$H_\phi = \frac{1}{\mu}(\nabla \times \overline{A})\phi$$

But $$(\nabla \times \overline{A})\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \theta}\right]$$

Now the current element is placed along z-axis,

$$\therefore\ A_\theta = -A_z \sin\theta ,$$

$$A_r = 0$$

$$\therefore\ (\nabla \times \overline{A})_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(r)(-A_z \sin\theta)\right]$$

Substituting value of $(\nabla \times \overline{A})_\phi$ in equation (15), we get,

$$H_\phi = \frac{1}{\mu}\left[\frac{1}{r}\frac{\partial}{\partial r}(-r A_z \sin\theta)\right]$$

Substituting value of A_z we get,

$$H_\phi = \frac{1}{\mu}\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{-r\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right]\sin\theta\right\}\right]$$

$$\therefore\ H_\phi = \frac{-I_m}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]\frac{d}{dr}\ [e^{-j\beta r}]$$

$$\therefore\ H_\phi = \frac{-I_m}{2\pi\beta r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right][(e^{-j\beta r})(-j\beta)]$$

$$\therefore\ \boxed{H_\phi = \frac{j I_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]}$$

The magnitude of the magnetic field strength for the radiation field of a half wave dipole or quarter wave monopole is given by,

$$|H_\phi| = \frac{I_m}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

The electric field strength is related to the magnetic field strength by the relation,

$$\frac{E_\theta}{H_\phi} = \eta$$

For free space, $\eta = \eta_0 = 120\pi$

$$\therefore \quad E_\theta = (120\pi) H_\phi$$

Substituting value of H_ϕ from equation (18), we get,

$$E_\theta = (120\pi)\left\{\frac{j I_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]\right\}$$

$$\therefore \quad E_\theta = \frac{j60 I_m e^{-j\beta r}}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

The magnitude of the electric field strength for the radiation field of a half wave dipole or a quarter wave monopole is given by

$$|E_\theta| = \frac{60 I_m}{r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

The average value of the power is half of the maximum value.

$$\therefore \quad P_{avg} = \frac{P_{max}}{2} = \frac{15 I_m^2}{\pi r^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2$$

The effective on r.m.s. current is related to the maximum current by the relation given by,

$$I_{rms} = \frac{I_m}{\sqrt{2}} \text{ i.e. } I_m = \sqrt{2}\, I_{rms}$$

Hence the average power can be expressed interms of r.m.s. current as,

$$P_{avg} = \frac{15(\sqrt{2}\,I_{rms})^2}{\pi r^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2$$

$$\therefore \quad \boxed{P_{avg} = \frac{30\,I_{rms}^2}{\pi r^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2}$$

The total power radiated through the hemispherical surface of radius r is given by,

$$\text{Power radiated} = \oint_{surface} P_{avg}\, ds$$

where ds = element of area of the spherical shell = $2\pi r^2 \sin\theta\, d\theta$

Substituting the value of P_{avg} and ds we get,

$$\text{Power radiated} = \oint_{surface} \frac{30\,I_{rms}^2}{\pi r^2}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2 (2\pi r^2 \sin\theta\, d\theta)$$

$$\boxed{= 60\,I_{rms}^2 \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\sin\theta\right)}{\sin\theta}\, d\theta}$$

We can find the integral function value by using Simpson's or trapezoidal rule as

θ	0deg.	5deg	10 deg	15 deg	20 deg	25 deg	30 deg	35 deg	40 deg	45 deg
$\frac{\cos^2\left(\frac{\pi}{2}\sin\theta\right)}{\sin\theta}\, d\theta$	0	0	0.003	0.011	0.028	0.050	0.086	0.138	0.201	0.280

θ	50deg	55 deg	60 deg	65 deg	70 deg	75 deg	80 deg	85 deg	90 deg
$\frac{\cos^2\left(\frac{\pi}{2}\sin\theta\right)}{\sin\theta}\, d\theta$	0.369	0.468	0.578	0.688	0.788	0.875	0.942	0.980	1.00

$\int_0^{\pi/2} cos2(\left(\frac{\pi}{2}\right) cos\theta)/sin\theta$= (π/2)(1/8)[(1.00+0)/2+$\sum_0^{85} cos2(\left(\frac{\pi}{2}\right) cos\theta)/sin\theta$]

=0.609

Hence the radiated power interms of the effective or r.m.s. current is given by,

$$\text{Power radiated} = 60\, I_{rms}^2 (0.609) = 36.54\, I_{rms}^2$$

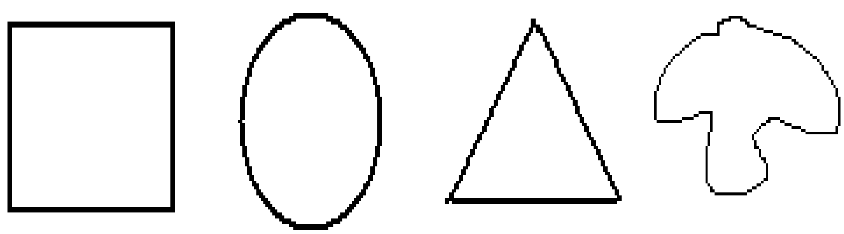
Hence the radiation resistance of the quarterwave monopole is

$$R_{rad} = 36.5\ \Omega$$

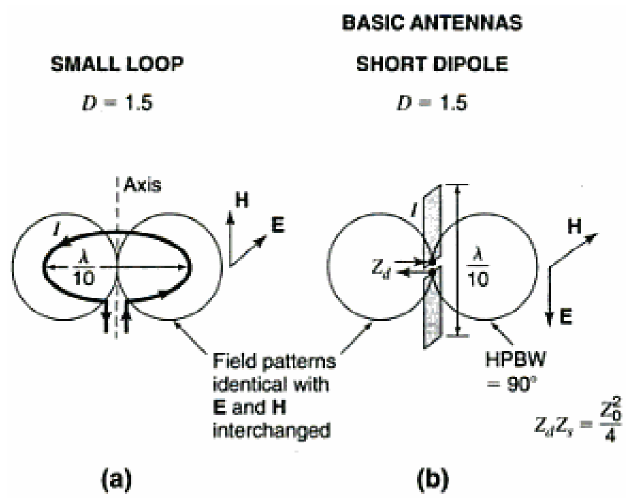
For a half-wave dipole, the power is radiated through a complete spherical surface. Hence for the same current the power radiated is double of that radiated by the quarterwave monopole. Hence the radiation resistance of the half wave dipole is twice of the radiation resistance of the quarter wave monopole. Hence the radiation resistance for half wave dipole is

$$\therefore \quad R_{rad} = 73\ \Omega$$

Loop antennas: A loop antenna can be formed by joining the two ends of a long straight wire or conductor. Thus the loop can be of any shape like square (rectangle), circle, triangle etc… as shown below.

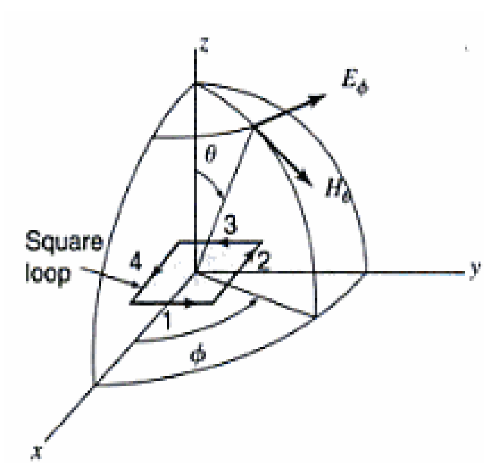


Generally square loop will be considered for the analysis purpose. Any circle, triangle or any other shaped loop can be converted into a square loop of having the same area. For example a circle of radius, r can be converted into a square loop of side,' a' such that $a^2=\pi r^2$.The small horizontal loop antenna shown below may be regarded as the magnetic counter part of the short dipole. Both loop and dipole have identical field patterns but with electric and magnetic fields interchanged. The horizontal loop is horizontally polarized and the vertical loop is vertically polarized. Both small loop and small dipole have the same directivity. To qualify as a small loop or short dipole, the dimensions should be λ/10 or less.



It is assumed that the loop dimensions are small compared to the wavelength, it will be shown that the far field patterns of circular and square loops of the same area are the same.

Comparison of far fields of a small loop and a small dipole: If the loop is placed at the origin of a spherical coordinate system as shown in the below figure its far field can be calculated by treating this loop as a combination of 4 short dipoles 1,2,3 and 4.



By doing so the comparison is made as shown in the below table.

Field	Electric Dipole	Loop
Electric	$\frac{E\theta = J60\Pi\ I\ dl\ sin\theta}{r}(\frac{L}{\lambda})$	$\frac{E\varphi = 120\Pi 2\ I\ dl\ sin\theta}{r}(\frac{A}{\lambda 2})$
Magnetic	$\frac{H\varphi = J\ I\ dl\ sin\theta}{2r}(\frac{L}{\lambda})$	$\frac{H\theta =\ \Pi I\ dl\ sin\theta}{r}(\frac{A}{\lambda 2})$

Where L is the length of the short dipole and A is the area of the small square loop. The presence of the operator j in the dipole expressions and its absence in the loop equations indicate that the fields of dipole and loop are in quadrature time-phase. The current I being in the same phase in both the dipole and loop.

Radiation resistance of Small Loops: The radiation resistance of small loop antennas can be found similar to the small dipoles i.e. the pointing vector is integrated over a sphere yielding the total power P_t radiated. This power is then equated to the square of the effective current on the loop times the radiation resistance, R_r. By doing so it is given by

Small loop

$$R_r = 31{,}171\left(\frac{A}{\lambda^2}\right)^2 = 197C_\lambda^4 \quad (\Omega)$$

$$\text{or} \quad R_r \simeq 31{,}200\left(\frac{A}{\lambda^2}\right)^2 \quad (\Omega)$$

Radiation resistance

Where A is the area of the loop and $C_\lambda=2\pi r/\lambda$.(r is the radius of the circular loop).This is the radiation resistance of a small single-turn loop antenna, Circular or square with uniform in phase current. The radiation resistance of a small loop of one or more turns is given by

Small loop $$R_r = 31{,}200\left(n\frac{A}{\lambda^2}\right)^2 \quad (\Omega)$$ *Radiation resistance*

Where n=no. of turns. The radiation resistance of a large loop can also be found as

Problems

2.1). How much power will an antenna having a radiation resistance of 50Ω radiate when it is fed by 20A of current?

Sol: Given data: I=20A

R_r = 50 Ω

Required data: P_t=?

We know that Radiated power, $P_t=I^2R_r$. = $(20)^2 50$=20000 watts.

2.2). What is the radiation resistance of an antenna which radiates 5KW when it draws 15A of current?

Sol: Given data: P_t =5KW

I=15A

Required data: R_r =?

We know that Radiated power, $P_t=I^2R_r$. or $R_r = P_t / I^2$=5000/(15X15)=22.2 Ω.

2.3). An antenna having radiation resistance of 75 Ω is radiating 10KW of power. How much current will flows into the antenna?

Sol: Given data: R_r =75 Ω

P_t =10KW

Required data: I=?

We know that Radiated power, $P_t=I^2R_r$. or $I^2= P_t / R_r$ or I=11.547A.

2.4). Calculate the radiation resistance of a current element whose overall length is λ/50?

Sol: Given data: dl= λ/50

Required data: R_r =?

We know that $80\pi^2\left(\frac{dL}{\lambda}\right)^2$

$= 80\pi^2 (\lambda / 50\lambda)^2=0.315\Omega$.

2.5). Calculate the radiation resistance and efficiency of a current element whose overall length is λ/50 and loss resistance is 1.5Ω?

Sol: Given data: dl= λ/50

R_l=1.5Ω

Required data: =?

We know that $=R_r/(R_r +R_l)$

And $R_r = 80\pi^2\left(\frac{dL}{\lambda}\right)^2$

$= 80\pi^2 (\lambda / 50\lambda)^2=0.315\Omega$.

= 0.315/ (0.315+1.5)=0.173=17.3%

2.6). At what distance in terms of wave lengths is the radiation component of magnetic field three times the induction component. At what distance is it 50 times?

Sol: Given data: (a) 3 times of induction field=radiation field.

(b) 50 times of induction field=radiation field.

R_l=1.5Ω

Required data: (a) r (3 times) =?

(b) r (50 times) =?

(a)We know that $H_\phi = \frac{I\,dL\sin\theta}{4\pi}\left[\frac{-\omega\sin\omega t'}{rv} + \frac{\cos\omega t'}{r^2}\right]$

It gives that $3[\frac{I\ dl\ sin\theta}{4\pi}(\frac{cos\omega t\prime}{r2})] = [\frac{I\ dl\ sin\theta}{4\pi}(\frac{-\omega sin\omega t\prime}{rv})]$

$\frac{3}{r2}=\frac{\omega}{rv}$

3/r =ω/v =2πf/fλ =2π/λ

2πr=3λ

r =3λ/2π=0.477 λ.

(b) Similarly $50/r^2$=ω/rv

50/r= ω/v=2πf/fλ

50 λ=2πr

r =50 λ=2π=7.96λ.

2.7). The radiation resistance of an antenna is 80 Ω. And loss resistance is 10 Ω. Find its directivity if power gain is 20?

Sol: Given data: R_r = 80 Ω

R_l=10Ω

G =20

Required data: D=?

We know that η=G/D=$\frac{Rr}{Rr+Rl}$=80/(80+10)=0.89

D=G/ η=20/0.89=22.47

2.8). An antenna has a loss resistance of 10 Ω. Power gain of 20 and directivity 22.Calculate radiation resistance?

Sol: Given data: D = 22

R_l=10Ω

G =20

Required data: R_r=?

We know that η=G/D=$\frac{Rr}{Rr+Rl}$

20/22=$\frac{Rr}{Rr+10}$

$22R_r=20R_r+200$

$2\,R_r=200$

$R_r=100\ \Omega$.

2.9). Calculate the directivity of an isotropic antenna?

Sol: Given data: Isotropic antenna(which radiates energy in all the directions uniformly)

Required data: D(isotropic)=?

We know that $D=4\pi/\ \Omega_A$

Ω_A of isotropic antenna is 4π because the radiation pattern of isotropic antenna is a sphere and in a sphere there will be 4π steradians (solid angle).

i.e. D (isotropic)= $4\pi/\ \Omega_A=4\pi/4\pi=1$.

2.10). Calculate maximum effective aperture of an antenna which is operating at wave length of 5 meters and has a directivity of 75?

Sol: Given data: λ = 5 meters.

D=75

Required data: A_e=?

We know that: $A_e=D\,\lambda^2/4\pi=(75X5X5)/\ 4\pi=149.28$ Square meters.

2.11). A plane electromagnetic wave having a frequency of 20MHz has an average Poynting vector of $10W/m^2$. If the medium is lossless with relative permeability of 4 and relative permittivity of 5. Find (i) velocity of propagation (ii) wave length (iii) Impedance of the medium?

Sol: Given data: f=20MHz

$P = 10W/m^2$

$\mu_r= 4$ and $\epsilon_r =5$

Required data: (i) v=? (ii) λ=? And (iii) η=?

We know that: (i) $v=1/\sqrt{\mu\epsilon} =1/\sqrt{\mu r\epsilon r\mu 0\epsilon 0} = 1/\sqrt{\mu r\epsilon r}\ 1/\sqrt{\mu 0\epsilon 0} =c/1/\sqrt{\mu r\epsilon r}$

$= (3x10^8)/\sqrt{4x5} =0.67x10^8$m/s.

(ii) v = f λ, $\lambda =v/f =(0.67x10^8)/(20x10^6) = 3.35$m

(iii) η or $z_0= 1/\epsilon c = 1/\epsilon_0\epsilon_r(0.67x10^8) = 476.63\Omega$.

Introduction: Antenna array is one of the common methods of combining the radiations from a group or array of similar antennas or elements. The total field produced by an antenna array is the vector sum of the fields produced by the individual antennas of the array system.

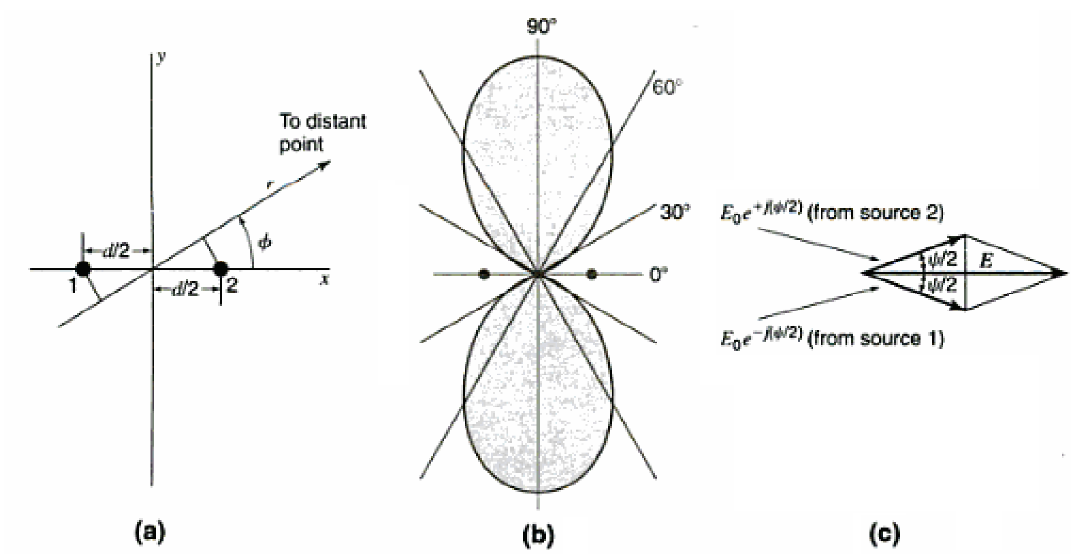
An antenna array is said to be linear, if the individual antennas of the array are equally spaced along the straight line. A uniform linear array is one in which the elements are fed with a current of equal amplitude with uniform progressive phases along the line.

Point sources: A point source can be defined as a minimum volume element ideally zero volume and capable of radiating the energy to a distant point. An isotropic point source is one which is able to radiate the energy in all the directions uniformly. That is the radiation pattern of isotropic point source is spherical.

Array of two isotropic point sources: Let us consider two isotropic point sources are along a straight line. In order to see the radiation at a distant point, P there exists 5 different cases. They are

1. Two isotropic point sources of same amplitude and same phase.
2. Two isotropic point sources of same amplitude and opposite phase.
3. Two isotropic point sources of same amplitude and quadrature phase.
4. Two isotropic point sources of same amplitude and any phase (General case).
5. Two isotropic point sources of any amplitude and any phase (Most General case).

1. Two isotropic point sources of same amplitude and same phase: Let the two point sources 1 and 2 be separated by a distance, d and located symmetrically with respect to the origin of the coordinate system as shown in the below figure (a).



The angle φ is measured counter clock wise from the positive x-axis. The origin of the coordinates is taken as the reference for phase. Then at a distant point, P in the φ direction, the field from source 1 is retarded by $\frac{dr\ cos\varphi}{2}$. While the field from source 2 is advanced by $\frac{dr\ cos\varphi}{2}$. Where d_r is the distance between the sources expressed in radians. That is $d_r = \frac{2\pi d}{\lambda}$ =βd. The total field at a large distance, r in the φ direction is then $E=E_0\, e^{-j\Psi/2}+E_0\, e^{j\Psi/2}$ Where Ψ= $d_r cos\varphi$ and E_0 is the amplitude. The first term in the above equation is the field component due to source 1 and second term is the field component due to source 2. This equation can also be written as

$$E= 2E_0\, \frac{e^{j\Psi/2}+e^{-j\Psi/2}}{2}$$

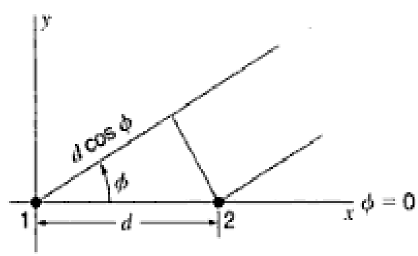
$$E= 2E_0 \cos (\Psi/2)$$

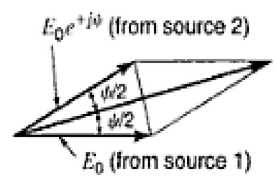
$$E= 2E_0 \cos \left(\frac{dr\ cos\varphi}{2}\right)$$

This result can also be obtained from vector diagram shown in figure(c). For the particular case take $2E_0$ =1 and d=λ/2 i.e. dr = $\frac{2\pi\lambda}{2\lambda}$ = π. Putting these conditions in the above equation gives

This field pattern of E verses φ as expressed in this equation is represented in figure (b). This pattern is a bidirectional figure-of-eight with maximum along Y-axis. The space pattern is doughnut shaped, being a figure of revolution of this pattern around X-axis.

The same pattern can also be obtained by locating source 1 at the origin of the coordinates and source 2 at a distance, d along the positive X-axis as shown in below figure (a). The vector diagram of the fields is shown in figure (b).





2. Two isotropic point sources of same amplitude and opposite phase: This case is identical with the first case except that the two sources are in opposite phase. Let the sources be located as in above figure (a) of first case, then the total field in the direction φ at a large distance r is given by $E=E_0\, e^{-j\Psi/2}- E_0\, e^{j\Psi/2}$ = $2jE_0 \sin (\Psi/2)$ = $2jE_0 \sin \left(\frac{dr\ cos\varphi}{2}\right)$. Putting $2jE_0$ =1 and considering the particular case of d=λ/2. The above equation becomes as

$$E = \sin \left(\frac{\pi}{2}\cos\varphi\right).$$

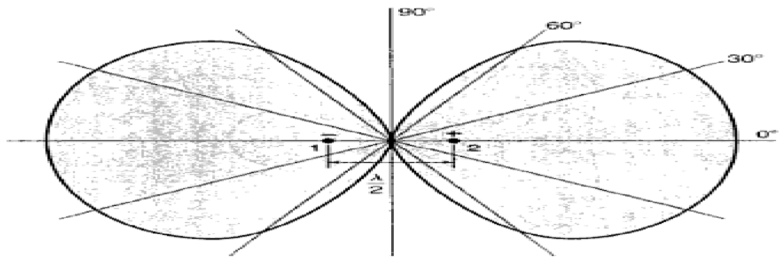
The directions φ_{max}. Of maximum field are obtained by setting the $\frac{\pi}{2}\cos\varphi_{max} = \pm(2k+1)\frac{\pi}{2}$.

Where k=0, 1, 2, …. For k=0, $\cos\varphi_{max} = \pm 1$ and $\varphi_{max.} = 0^0$&180^0

The null directions φ_0 are given by $\frac{\pi}{2}\cos\varphi_0 = \pm k\pi$. For k=0, $\varphi_0 = \pm 90^0$.

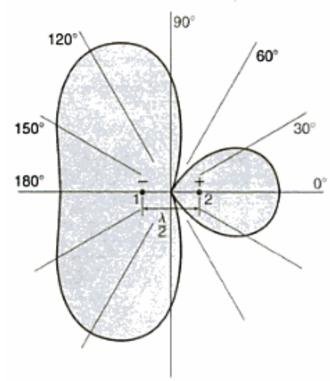
The half power directions are given by $\frac{\pi}{2}\cos\varphi_{HP} = \pm(2k+1)\frac{\pi}{4}$. For k=0, $\varphi_{HP} = \pm 60^0, \pm 120^0$.

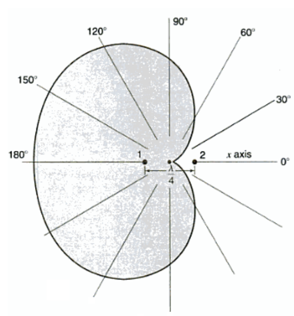
Finally the field pattern for the equation E = sin $(\frac{\pi}{2}\cos\varphi)$. will be as shown in the below figure.



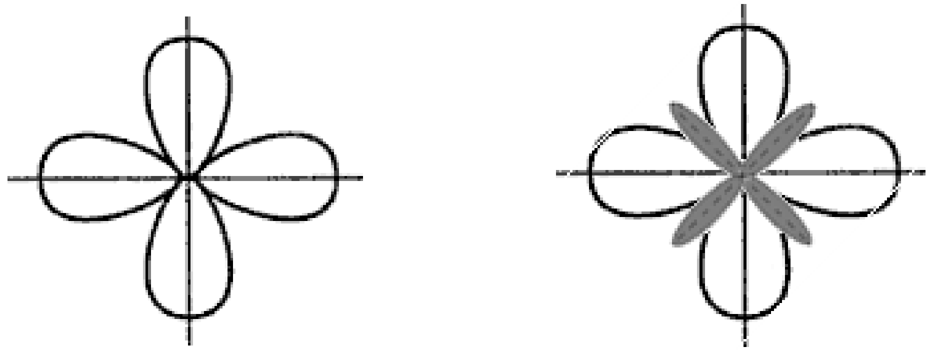
The pattern is relatively figure of eight with the maximum field in the same direction as the line joining the sources (X-axis). The space pattern is a figure of revolution of this pattern around the X-axis. The two sources in this case may be described as end-fire array. Whereas in case has maximum field normal to the line joining the sources may be described as the broad side array.

3. Two isotropic point sources of same amplitude and quadrature phase: Let the two point sources be located as in figure (a) of case 1. Taking the origin of the coordinates as the reference for phase, Let source 1 be retarded by 45^0 and source 2 advanced by 45^0. Then the total field in φ direction at a large distance, r is given by $E = E_0\, e^{-(\frac{j\Psi}{2}+\frac{\pi}{4})} + E_0\, e^{(\frac{j\Psi}{2}+\frac{\pi}{4})} = 2E_0 \cos(\frac{\pi}{4} + \frac{\Psi}{2})$. Putting $2E_0=1$ and d=λ/2 we get E = cos $(\frac{\pi}{4} + \frac{\pi}{2}\cos\varphi)$. The field pattern given by above equation is represented in the below figure. The space pattern is a figure-of-revolution of this pattern around X-axis. Most of the radiation is in 2nd and 3rd quadrants. The directions φ_{max}. Of maximum field are obtained by setting $\frac{\pi}{4} + \frac{\pi}{2}\cos\varphi_{max} = k\pi$. Where k=0, 1, 2……For k=0, $\frac{\pi}{2}\cos\varphi_{max} = \frac{-\pi}{4}$ and $\varphi_{max.} = 120^0$ and 240^0. If the spacing between the sources is reduced to λ/4 then E = cos $(\frac{\pi}{4} + \frac{\pi}{4}\cos\varphi)$.The field pattern for this case is shown in below figure. It is a cardiod-shaped unidirectional pattern with maximum field in the negative X-direction. The space pattern is a figure-of-revolution of this pattern around the X-axis. In this case the Directivity is increasing in the negative X-direction and this case may be described as increased end-fire array or "Hansen-Woodyard" array after the names of the Scientists who worked for the description of this array.



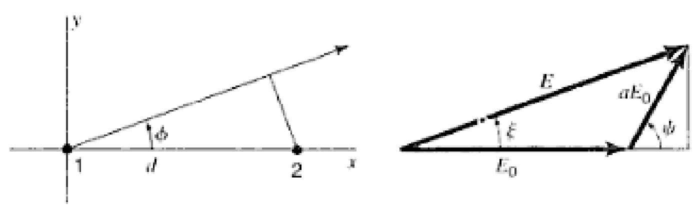


Similarly if d is increased between the sources as d = λ and d = 2λ the field patterns will be as shown below.



4. Two isotropic point sources of same amplitude and any phase (General case): Let us consider the case of two isotropic point sources of equal amplitude but of any phase difference δ. Now the total phase difference, Ψ between the fields from source 1 and source 2 at a distant point in φ direction is given by $\Psi = d_r \cos\varphi + \delta$. Taking the source 1 as reference for phase, the positive sign in this equation indicates that the source 2 is advanced in phase by the angle δ. A minus sign would be used to indicate a phase retardation. If instead of referring the phase to source 1, it is referred to the center point of the array, The phase of the field from source 1 is given by –Ψ/2 and that from source 2 by +Ψ/2. The total field is then $E=E_0\, e^{-j\Psi/2}+E_0\, e^{j\Psi/2}= 2E_0 \cos(\Psi/2)$.Putting $2E_0 =1$, $E= \cos(\Psi/2)$. Where $\Psi = d_r \cos\varphi+\delta$.In this if $\delta=0^0$ it is nothing but case 1, if $\delta=180^0$ it is case 2 and if $\delta=90^0$ it is case 3.

5. Two isotropic point sources of any amplitude and any phase (Most General case):



(a) (b)

Let the sources be situated as shown in above figure (a) with source 1 at the origin. Assume that source 1 has the larger amplitude E_0 and source 2 of amplitudeaE_0 ($0\leq a\leq 1$) at large distance, r. Then from vector diagram of above figure (b), the magnitude of total field E is given by $E= E_0 \sqrt{a(1+a\,cos\Psi)^2+(a\,sin\Psi)^2}$ and its phase is $\tan^{-1}(\frac{a\,sin\Psi}{1+a\,cos\Psi})$.Where $\Psi = d_r \cos\varphi+\delta$.

Principle of Multiplication of patterns: Multiplication of patterns or simply pattern multiplication is stated as follows:" The total field pattern of an array of non-isotropic but similar sources is the multiplication of the individual source pattern and the pattern of array of isotropic point sources (group pattern) each located at the phase center of individual source and having the relative amplitude and phase. Whereas the phase pattern is the sum of the phase patterns of individual source and that of the array of isotropic point sources". That is mathematically

$$E = \underbrace{f(\theta,\phi)F(\theta,\phi)}_{\text{Field pattern}}\ \underbrace{\angle f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{Phase pattern}}$$

where

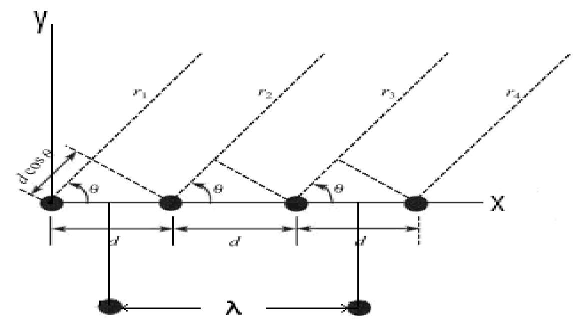
$f(\theta, \phi)$ = field pattern of individual source

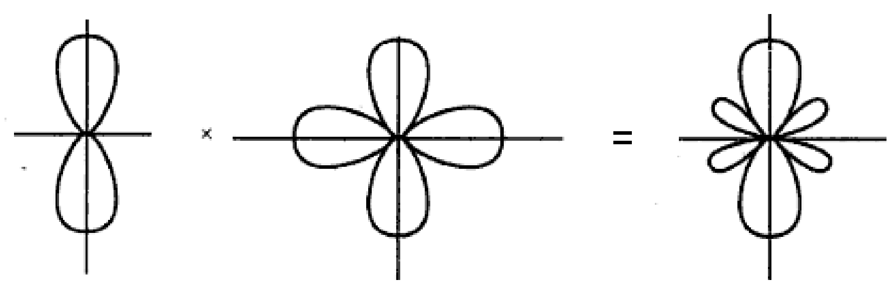
$f_p(\theta, \phi)$ = phase pattern of individual source

$F(\theta, \phi)$ = field pattern of array of isotropic sources

$F_p(\theta, \phi)$ = phase pattern of array of isotropic sources

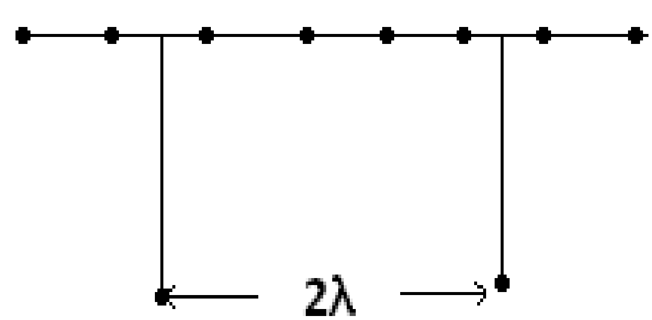
Radiation pattern of 4-isotropic elements fed in phase and spaced λ/2 apart: Let 4-isotropic point sources are placed as shown in below figure. To find the total field pattern of this array we can use the principle of Multiplication of patterns. From the figure the individual or source or unit pattern is the pattern of two isotropic point sources spaced λ/2 apart whose pattern is given by broad side array pattern as shown in below figure (a). And the group pattern is an array of two isotropic point sources separated by a distance of λ apart. Whose pattern is given by below figure (b). Multiplication of these two patterns is given by in figure (c).

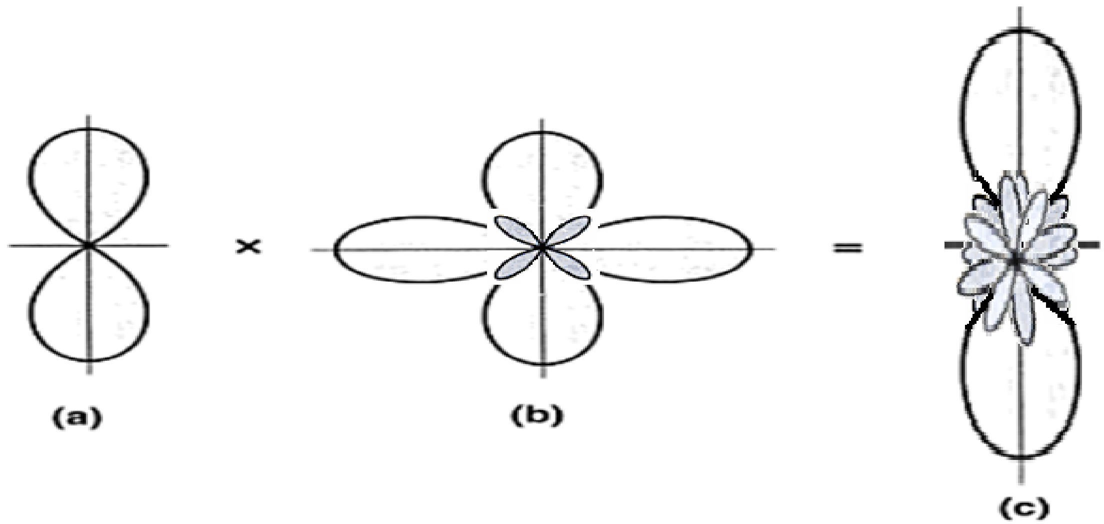




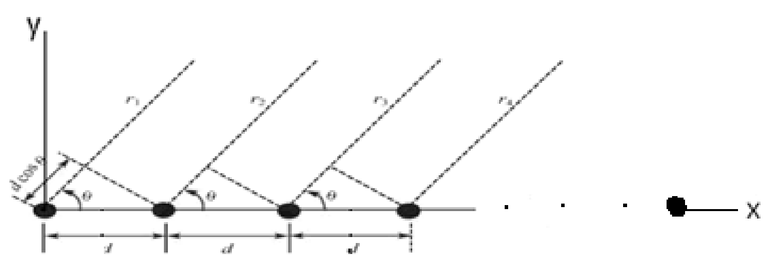
(a) (b) (c)

Radiation pattern of 8-isotropic elements fed in phase and spaced λ/2 apart: Let 8-isotropic point sources are placed as shown in below figure. To find the total field pattern of this array we can use the principle of Multiplication of patterns. From the figure the individual or source or unit pattern is the pattern of 4 isotropic point sources spaced λ/2 apart and Field pattern of this source array is shown in below figure (a). (i.e. figure (c) of previous case). And the group pattern is an array of two isotropic point sources separated by a distance of 2λ apart. Whose pattern is given by below figure (b). Multiplication of these two patterns is given by in figure (c).





Linear array of n-isotropic point sources of equal amplitude and spacing: Let n no. of isotropic point sources are placed along the X-axis with equal spacing, d as shown in below figure.



Now we have to calculate the pattern of this linear array of n isotropic point sources of spacing, d and amplitude, E_0. The total far field at a distant point, P is obtained by adding vectorally the fields of individual sources as $E_t = E_0 + E_0 e^{j\Psi} + E_0 e^{j2\Psi} + E_0 e^{j3\Psi} + \ldots\ldots\ldots\ldots. + E_0 e^{j(n-1)\Psi}$

$$E_t = E_0\,(1 + e^{j\Psi} + 2 + \ldots\ldots\ldots\ldots. + e^{j(n-1)\Psi}) \_\_\_\_\_\_\_\_\_\_ (1)$$

Where $\Psi = d_r \cos\varphi + \delta$. Multiplying both sides with $e^{j\Psi}$, we get

$$E_t e^{j\Psi} = E_0\,(e^{j\Psi} + e^{j2\Psi} + e^{j3\Psi} + \ldots\ldots\ldots\ldots. + e^{jn\Psi}) \_\_\_\_\_\_\_\_ (2)$$

Subtracting equation 2 from 1 or using the geometric series formula $\frac{1-a^n}{1-a}$ where $a = e^{j\Psi}$., we get

$$E_t = E_0 \left[\frac{1-e^{jn\Psi}}{1-e^{j\Psi}} \right]$$

$$E_t = E_0 \left[\frac{1-e^{jn\Psi/2} e^{jn\Psi/2}}{1-e^{j\Psi/2} e^{j\Psi/2}} \right]$$

$$E_t = E_0 \frac{-e^{jn\Psi/2}}{-e^{j\Psi/2}} \left[\frac{e^{jn\Psi/2} - e^{-jn\Psi/2}}{e^{j\Psi/2} - e^{-j\Psi/2}} \right]$$

$$E_t = E_0\, e^{j(n-1)\Psi/2} \left[\frac{\sin(n\Psi/2)}{\sin(\frac{\Psi}{2})} \right]$$

$$E_t = E_0 \left[\frac{\sin(n\Psi/2)}{\sin(\frac{\Psi}{2})} \right] e^{j\varphi} \quad [\text{Let } \varphi = \frac{(n-1)\Psi}{2}]$$

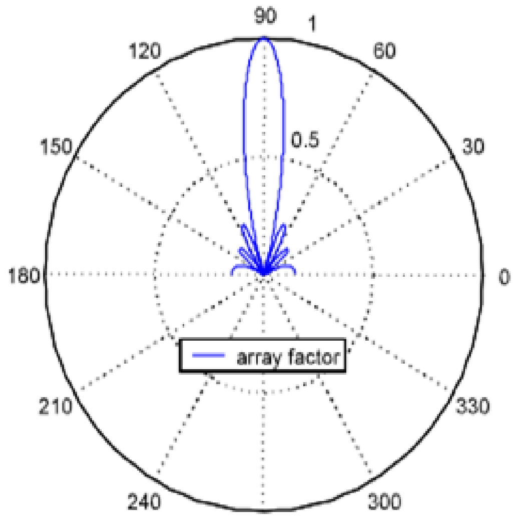
This is the equation of total far field pattern of linear array of n-isotropic point sources where source 1 as reference point for phase. If the reference point is shifted to the center of the array from the source or origin then phase angle $\varphi = \frac{(n-1)\Psi}{2}$ is automatically eliminated because we are having n/2 no. of delayed elements in the left hand side and same n/2 no. of advanced elements in the right hand side results finally in zero phase. Then

Further instead of isotropic sources if non-isotropic but similar sources are considered then in the above equation E0 will represent individual or source or unit pattern and $\frac{\sin(n\Psi/2)}{\sin(\frac{\Psi}{2})}$ is the group pattern or secondary pattern or array factor.

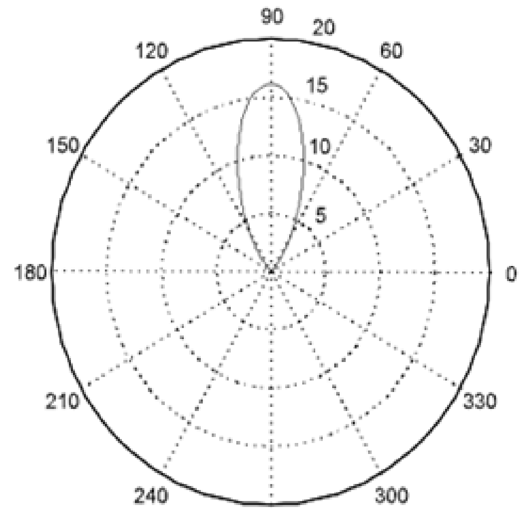
Binomial array: The disadvantage of linear arrays is that there exist minor lobes along with the major lobes. These minor lobes can be eliminated in non-linear arrays i.e. by changing the amplitudes of the center elements which are the sources for the occurrence of minor lobes in the total or resultant pattern. The Binomial array is one of such non-linear arrays invented by Stone in 1929. In this array the current amplitudes of the array are proportional to the coefficients of the Binomial series. Which can be found using the Pascal's triangle in which any internal integer is the sum of the above adjacent integers as shown below.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

However, the elimination of secondary or minor lobes takes place at the cost of Directivity. Half power beam width of Binomial array is more than that of the linear array for the same length of the array. Thus in linear array minor lobes appear but major lobe is sharp and narrow whereas in Stone's array width of the beam widens but without minor lobes as shown in below figure.



n = 5, $d = 0.5\lambda$ Linear Array

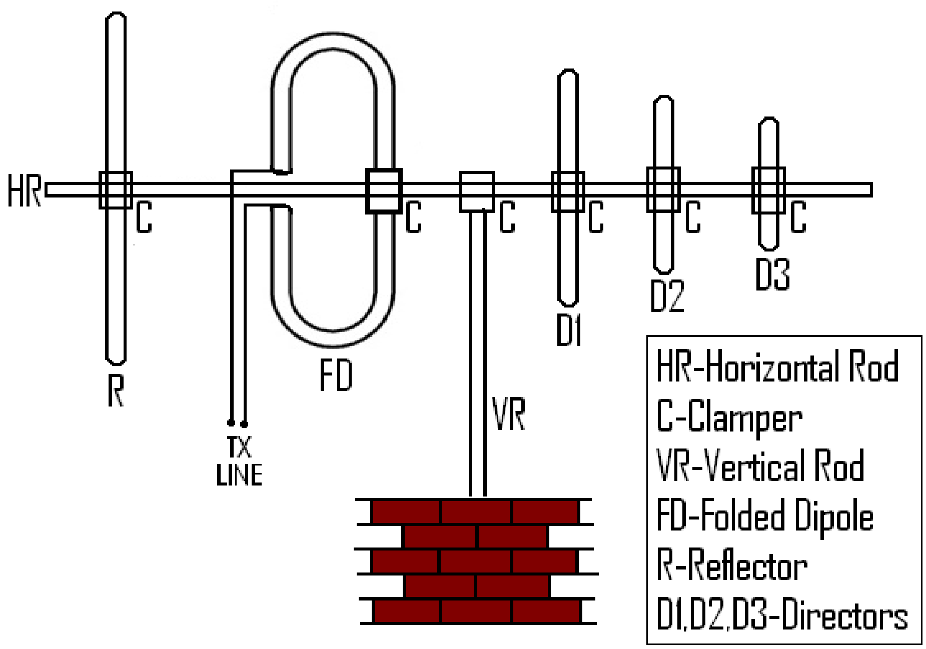


n = 5, $d = 0.5\lambda$ Binomial Array

UNIT-4 VHF, UHF & MW ANTENNAS-1

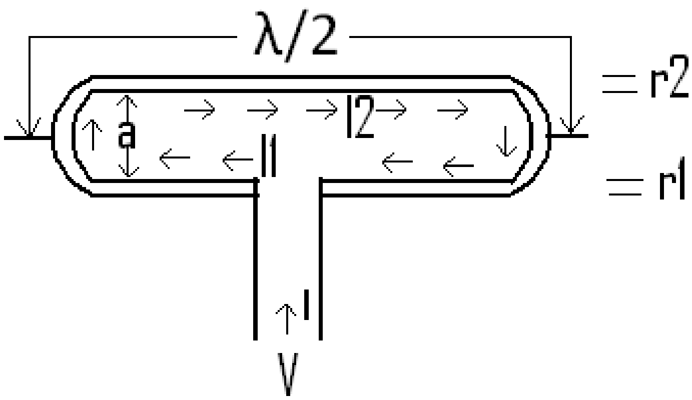
Introduction: VHF, UHF and Micro Wave antennas are those antennas which are operating between the frequency range of 30 to 300 MHz, 300 to 3000 MHz and above 2000MHz to several thousands of MHz respectively. That is all the antennas available can be divided into different categories depending on their frequency range of operation.

Yagi-Uda antenna: Yagi-Uda or simply Yagi antenna is a VHF and UHF range antenna named after S.Uda and H.Yagi for the reception of Television signals. The Yagi-Uda antenna consists of a driven element, a reflector and one or more directors i.e. it is an array of driven element and one or more parasitic elements i.e. passive elements which are not connected directly to the transmission line but electrically coupled. Where as the driven element which is a folded dipole is connected directly to the transmission line. A practical Yagi-Uda antenna used for the reception of TV signals is shown below.



As shown in the figure the Yagi-Uda antenna is consisting of three elements namely the folded dipole, reflector and directors. The array elements are arranged with the help of Horizontal, vertical rods along with the clampers. Now we will see about the remaining actual elements of Yagi-Uda parasitic array.

Folded Dipole: The folded dipole is one in which two half-wave dipoles one continuous and the other spitted at the center have been folded and joined together in parallel at the ends. The spitted dipole is fed at the center by a balanced transmission line. The two dipoles therefore will have the same voltages at their ends as shown in the below figure.



If the radii r_1 and r_2 of two dipoles are equal, then equal currents flow. If I_1 and I_2 are currents flowing in the two dipoles, then the voltage at the ends of the two dipoles is given by V/2 and it can be expressed asV/2 = $i_1z_{11}+i_2z_{22.}$ Where Z_{11} is the self impedance of dipole1 and Z_{12} is the mutual impedance between the two dipoles. But for dipoles of equal radii, $I_1 = I_2$. Then V/2= $I_1 (Z_{11}+Z_{12})$. If the two dipoles are very close to each other i.e. the spacing 'a' is of the order of λ/100 then Z12≈ Z11. Therefore

$$\frac{V}{2} = I_1 (Z_{11}+Z_{12})$$

$$\frac{V}{I1} = Z = 2^2(Z_{11}) = 4Z_{11} = 4X73 = 292\Omega.$$

Where Z_{11} is the radiation resistance of a half-wave dipole. This impedance Z =292Ω. Can be called as the input impedance or terminal impedance or radiation resistance of folded dipole antenna.

Similarly for a folded dipole of 3 elements (Tripole) we can write

$$\frac{V}{3} = I_1 (3\, Z_{11})$$

$$\frac{V}{I1} = Z = 3^2(Z_{11}) = 9X73 = 657\Omega.$$

Or generally we have $\frac{V}{n} = I_1 (n\, Z_{11})$

$$\frac{V}{I1} = Z\, n^2\, Z_{11} = n^2X73\ \Omega.$$

Where n is the number of half-wave dipoles. The impedance transformation can also be possible by making unequal radii of the two dipoles. Hence the formula in this case can shown as

$$Z = Z11(1+\frac{r2}{r1})^2 = 73(1+\frac{r2}{r1})^2$$

Where r_2 and r_1 are the radii of the dipoles in folded arrangement. If r2=2r1, then

$$Z = 73(1+\frac{2r2}{r1})^2\ 73x9 = 657\ \Omega.$$

Since the impedance transformation not only depends upon relative radii but also on the relative spacing and hence the formula according to Uda and Mushaike is given by

$$Z = Z_{11}\, [1+\frac{Log(\frac{a}{r1})}{Log(\frac{a}{r2})}]^2$$

The radiation resistance of folded dipole of equal radii is given by $R_r = 2^2 x73 = 292\ \Omega$. Shows that a two wire folded dipole can be fed directly with the conventional transmission line of impedance 300 Ω. Without any matching device. i.e. folded arrangement is placed in Yagi-Uda antenna as driven element for the purpose of impedance matching.

Parasitic elements: There are two parasitic elements used in Yagi-Uda array namely Reflector and Directors. These parasitic elements receive their excitation from the voltages induced in them by the current flow in the driven element. The parasitic element in front of driven element is known as director and its number may be more than one. Where the element in back of the driven element is known as reflector. The reflector is 5% more and director is 55 less in length than driven element.

The reflector having greater length than driven element will be inductive while elements of length less than driven element will be capacitive. Hence the phases of currents in reflectors will lag the induced voltage whereas in directors the currents will lead the induced voltage. The director adds fields of driven element in the direction away from the driven element. If more than one directors are employed, then each director will excite the next. Whereas the reflector adds up the fields of driven element in the direction from reflector towards driven element. Finally the radiation pattern of Yagi-Uda array is unidirectional along the directors. The directivity can be increased by increasing the no. of directors in the array.

Voltage and Current relations in Yagi-Uda array: The relations between voltages and currents of an Yagi-Uda antenna can be given by considering the general equations as

$$V_1 = I_1Z_{11}+I_2Z_{12}=I_3Z_{13}+ - - - +I_nZ_{1n}$$

$$V_2 = I_1Z_{21}+I_2Z_{22}=I_3Z_{23}+ - - - +I_nZ_{2n}$$

$$V_3 = I_1Z_{31}+I_2Z_{32}=I_3Z_{33}+ - - - +I_nZ_{3n}$$

-

-

-

$$V_n = I_1Z_{n1}+I_2Z_{n2}=I_3Z_{n3}+ - - - +I_nZ_{nn}$$

where $V_1,V_2,V_3 \ldots V_n$ are voltages applied to antenna numbers 1,2,3,…n

$I_1,I_2,I_3 \ldots I_n$ are currents flowing in antenna numbers 1,2,3,…n

$Z_{11},Z_{22},Z_{33} \ldots Z_{nn}$ are self impedances of antenna numbers 1,2,3,…n

$Z_{12},Z_{21},Z_{13} \ldots Z_{31}$ are the mutual impedances.

If the individual antennas are not excited, then corresponding voltages will be zero. Thus an antenna system involving parasitic elements, the voltages are zero in case of transmitting while in receiving case these are the voltages induced in each parasitic element by the electromagnetic wave.

Considering the simplest case with one driven element and one parasitic element we can write $V_1 = I_1Z_{11}+I_2Z_{12}$ and $V_2 = I_1Z_{21}+I_2Z_{22}$

Since $z_{12} = z_{21}$ and $V_2 = 0$ Being parasitic. The second equation becomes

$0 = I_1Z_{12}+I_2Z_{22}$ or $I_1Z_{12}= -I_2Z_{22}$ or $I_2 = -I_1 (\frac{Z12}{Z22})$

Therefore $V_1 = I_1Z_{11}- I_1 (\frac{Z12^2}{Z22})$

$V_1 = I_1(Z_{11}- \frac{Z12^2}{Z22})$

Or $I_1 \frac{V1}{(Z11-\frac{Z12^2}{Z22})}$

Now $I_2 = -I_1 (\frac{Z12}{Z22}) = \frac{-V1}{(Z11-\frac{Z12^2}{Z22})} (\frac{Z12}{Z22}) = \frac{-V1Z\,12}{Z11\,Z22-Z12^2}$

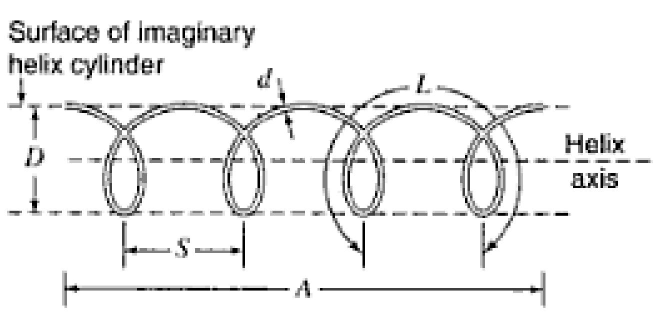
From equations of I_1 and I_2, the input impedances of driven and parasitic elements are given by

$$Z_1 = \frac{V1}{I1} = Z11 - \frac{Z12^2}{Z22} \text{ and } Z_2 = \frac{V2(orV1)}{I2} = Z12 - \frac{z11\,z22}{Z12}.$$

Helical Antenna: The Helical antenna was invented by John D. Krauss in the year 1946 by altering the dimensions of helix and conducting the experiment for each change in dimension of helix.

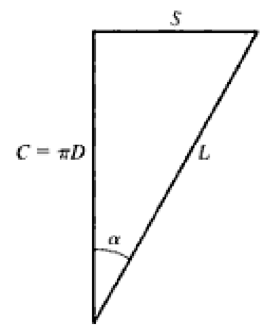
Helical geometry: The helix is a basic three dimensional geometric form. A helical wire on a uniform cylinder becomes a straight wire when unwound by rolling the cylinder on a flat surface. Viewed end on a helix projects as a circle. Thus a helix combines the geometric forms of a straight line, circle and a cylinder. In addition helix has handedness it can be either left or right-handed.

The following symbols are used to describe a helix as shown in below figure.

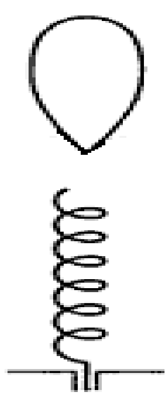


Where	D is	Diameter of helix (center to center).
	S is	Spacing between turns (center to center).
	L is	Length of one turn.
	A is	Axial length =ns.
	n is	No. of turns.
	d is	Diameter of the helix conductor or wire. And in addition
	c is	Circumference of helix = πD and
	α is	Pitch angle = $\tan^{-1}(\frac{S}{c}) = \tan^{-1}(\frac{S}{\pi D})$.

If one turn of a circular helix is unrolled on a flat surface, the relation between the spacing S, Circumference C, length L and pitch angle α is given by the below triangle. When the spacing is zero, then $\alpha=0^0$ and the helix becomes a loop on the other hand if the diameter is zero then $\alpha=90^0$. And the helix becomes a linear conductor.

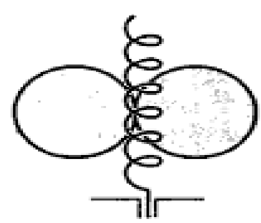


Modes of Helical antenna: Depending on the no. of turns considered the helix can be of monofilar, bifilar, quadrifiler, 8-lobed etc… By changing the geometry of helix and its orientation different types of radiation patterns can be obtained like unidirectional, bidirectional, 4-lobed and Omni directional also. Out of these the mostly used patterns are unidirectional and bidirectional. These two patterns can be obtained in two different modes of operations namely Axial mode and Normal mode of operations. In axial mode of operation the important dimension to be maintained is Axial Ratio (AR) as given by AR = $(\frac{2n+1}{2n})$. Where n is no. of turns in helix. In this axial mode of operation the maximum radiation is in the direction of helix axis as shown in below figure (a).



Axial mode

(a)

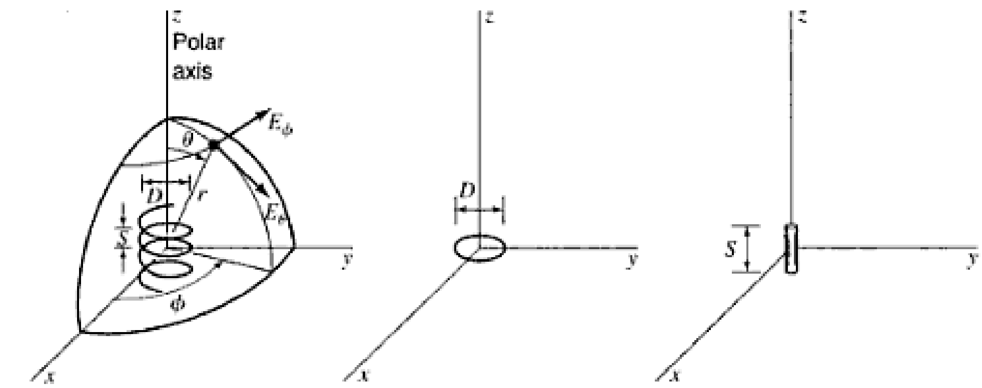


Normal mode

(b)

When the helix circumference is approximately a wave length the axial mode of operation is dominant which is unidirectional. But when the circumference is much smaller, the normal mode is dominant. This is bidirectional radiation pattern normal to the helix axis as shown in above figure (b).

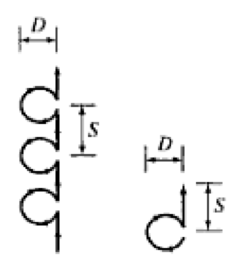
Consider a helix oriented with its axis is coincided with z-axis as shown in below figure (a).



(a) Helix **(b)** Loop **(c)** Dipole

If the dimensions are small (nL<<λ), the maximum radiation is in XY plane for a helix oriented as in figure (a) with zero field in the Z-direction. When the pitch angle $\alpha=0^0$, the helix becomes a loop as in above figure (b). When $\alpha=90^0$, the helix becomes a straight conductor as in above figure (c).

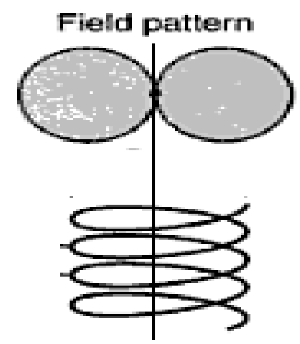
The loop and straight conductor being the limited cases of helix therefore a helix can be considered as a series connection of small loops and straight dipoles as in below figure (a). The diameter D of the loops is same as the helix diameter and length of the dipoles is same as the spacing between turns of the helix. Since the helix is small the far field pattern is independent of the no. of turns. Hence it is sufficient if we find the far field due to a single small loop and short dipole as in figure (b). The far field of this small loop has only E_ϕ component and is given by $E_\phi = \frac{120\pi^2 \, Idl \, Sin\theta}{r} \frac{A}{\lambda^2}$ ________________ (1)



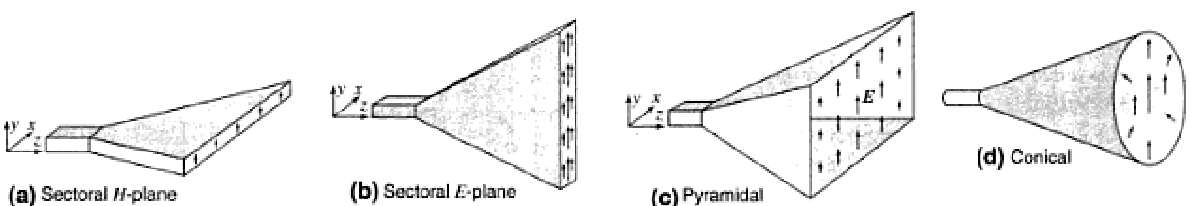
(a) **(b)**

Where A is area of the small loop $A = \frac{\pi D^2}{4}$. The far field of short dipole has only E_θ component and is given by $E_\theta = j \frac{60\pi \, Idl \, Sin\theta}{r} \frac{S}{\lambda}$ ________ (2).

Where S=L as the length of the dipole. The ratio of magnitudes of equations (1) and (2) gives the axial ratio i.e. AR = $\frac{|E\theta|}{|E\varphi|} = \frac{S\lambda}{2\pi A} = \frac{2S\lambda}{\pi^2 D^2}$. When $E_\phi = 0$, AR is infinite and the polarization is vertical, When $E_\theta = 0$, AR is zero and the polarization is horizontal. i.e. when $|E\theta| = |E\varphi|$; AR is unity then the polarization is a circle. i.e. when Πd = $\sqrt{2s\lambda}$ we get circular polarization. But in general as with the change in the pitch angle α, the overall polarization will be bidirectional as shown in figure.



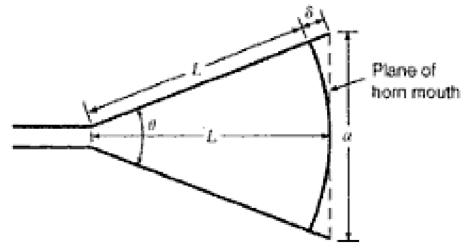
Horn Antennas: A Horn antenna may be considered as a flared out or opened out wave guide. A wave guide capable of radiating energy into open surface provided the same is excited at one end and opened at the other end. The radiation is much higher through wave guide than two wire transmission line. By opening the one end of wave guide the impedance matching will be poor with free space. In order to overcome this mouth of the wave guide is opened out which assumes the shape of electromagnetic horn. i.e. by terminating the opened end of wave guide as a horn improves impedance matching directly. If the opening or flaring is done only in one direction, then a sectorial horn is produced. Flaring in the direction of Electric & Magnetic vectors the sectorial E-plane horn and sectorial H-plane horn are obtained respectively as shown below.



If the flaring is done along both E and H planes of the rectangular wave guide, then a pyramidal horn is obtained. By flaring the walls of a circular wave guide, a conical horn will be obtained.

The function of the electromagnetic horn is to produce a uniform phase front with a larger aperture in comparison to wave guide and thus the directivity is greater.

Design equations of horn antennas: From the geometry of the horn as shown in the figure we have



$\cos\theta = \frac{L}{L+\delta}$ and $\tan\theta = \frac{h/2}{L} = \frac{h}{2L}$

$\theta = \tan^{-1}(\frac{h}{2L}) = \cos^{-1}\frac{L}{L+\delta}$ ....(1)

Where $\boldsymbol{\delta}$ is permissible phase angle variation as a fraction of 360^0 or it is the permissible path difference. From the above right angle triangle of the figure we can write $(L+\boldsymbol{\delta})^2 = L^2 + (h/2)^2$

$$\text{Or } L^2+\boldsymbol{\delta}^2+2L\,\boldsymbol{\delta} = L^2+(h^2/4)$$

If $\boldsymbol{\delta}$ is small $\boldsymbol{\delta}^2$ can be neglected in above equation $2L\boldsymbol{\delta} = h^2/4$, $L = \frac{h^2}{8\delta}$......... (2)

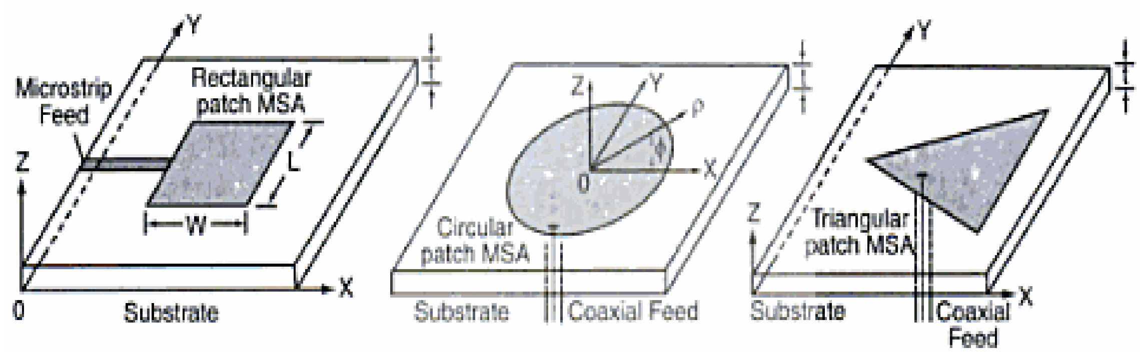
Equations1 and 2 the design equations for horn antennas. Equation 1 gives the optimum angle. The maximum directivity can be achieved with large flare angles for which $\boldsymbol{\delta}$ not exceeding its limited value (0.25 to 0.4).

The horn antennas are used at micro wave frequencies under the condition that power gain needed is moderate. For higher power gains the dimensions of horn becomes large, so the other antennas like Parabolic reflector or Lens antennas are preferred than horn antennas.

UNIT-5 VHF, UHF & MW ANTENNAS-2

Micro strip Antennas: It is a low profile, low gain, low bandwidth antenna. These are assigned with different names such as Patch antennas, Printed antennas, Micro strip Patch antennas or simply Micro strip antennas (MSA). These antennas are used where thickness and comfortability to the host surfaces are the key requirements. Since patch antennas can be directly printed onto a circuit board, these are becoming increasingly popular within the mobile phone market. They are low cost low profile and are easily fabricated. A practical Micro strip antenna was first introduced by Robert E.Munson.

Features: 1.Basically a patch antenna is a metal patch suspended over a ground plane. In its most basic form, a micro strip patch antenna consists of a radiating patch on one side of a dielectric substrate which has a ground plane on other side.



2. The simplest patch antenna uses a half wavelength long patch with a larger ground plane to give better performance.
3. A simple patch antenna radiates a linearly polarized wave. it is a narrow band antenna.
4. The Micro strip antenna may have a square, rectangular, circular, triangular or elliptical shape. Theoretically MSA's can be of any other continuous
5. Most generally used shape is Rectangular due to its simplest construction.
6. The size of a Micro strip antenna is inversely to its frequency.
7. The Micro strip antenna configuration employs a metallic patch which is positioned on the top surface of the dielectric substrate.
8. The Micro strip antenna is constructed on a thin dielectric sheet using a printed circuit board (PCB) and etching techniques.
9. Micro strip or patch antennas are popular at frequencies above 100MHz.
10. The radiation pattern of the patch is hemispherical in general. The typical beam area is half a hemi sphere or one fourth of a sphere i.e. π steradians. Hence the directivity of the

patch is given by $D = 4\pi/\Omega_A = 4\pi/\pi = 4$.or $D_{dbi} = 10 \log_{10}4 = 6.021$ dbi. (I stands for over isotropic).

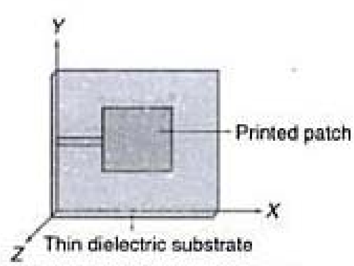
Advantages: Micro strip antennas are of

1. Light weight
2. Small size
3. Less volume
4. Can be molded easily to any shape.
5. Fabrication process is simple.
6. Production is easy.
7. Support both linear and circular polarizations.
8. Mechanically robust when mounted on rigid surfaces.
9. Fabrication process is compatible with micro wave monolithic integrated Circuit (MMIC) and Opto electronic integrated circuit (OEIC) technologies.

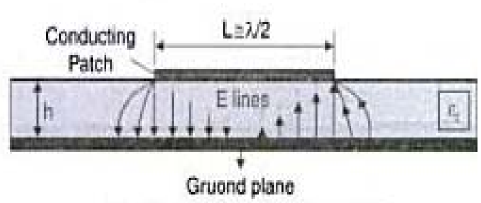
Limitations: Micro strip antennas suffer from a number of disadvantages as compared to conventional antennas. They are:

1. Low band width.
2. Low efficiency.
3. Low gain.
4. Low power handling.
5. Enhanced design complexity due to small size.
6. Suffer from effects of radiations from feeds and junctions.

Rectangular Micro strip antennas: The below figure (a) shows the basic structure of a rectangular micro strip antenna and figure (b) illustrates the side view of fig (a) in which the variation of electric field was shown.

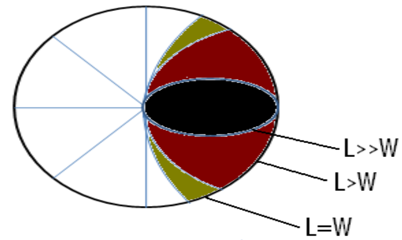


(a) Patch layout on a substrate



(a) Sinusoidal variation of E

The patch is of length L, width W and thickness h with permittivity ε_r. The thickness is not critically important. Typically h is much smaller than the wavelength of operation. The frequency of operation of the patch antenna is generally determined by length L. The critical frequency or center frequency f_c can be approximately given by $\frac{v}{2L\sqrt{\varepsilon r}} = \frac{v}{2L\sqrt{\mu 0 \varepsilon 0 \varepsilon r}}$ .Where v is the velocity of light; ε_0 and μ_0 are the permittivity and permeability of the dielectric substrate respectively. Generally the length L is maintained greater than the width, w. The radiation pattern of three such cases is shown in the below figure. The net magnitude of electric field at any point is a function of θ and ϕ as given by $E(\theta, \phi)=\sqrt{E\phi 2 + E\theta 2}$



Normally, the directivity of patch antennas is below 10dbi and the field for center fed rectangular patches is linearly polarized.

Characteristics of Micro strip antennas: Some of the most important characteristics of micro strip antennas are described below.

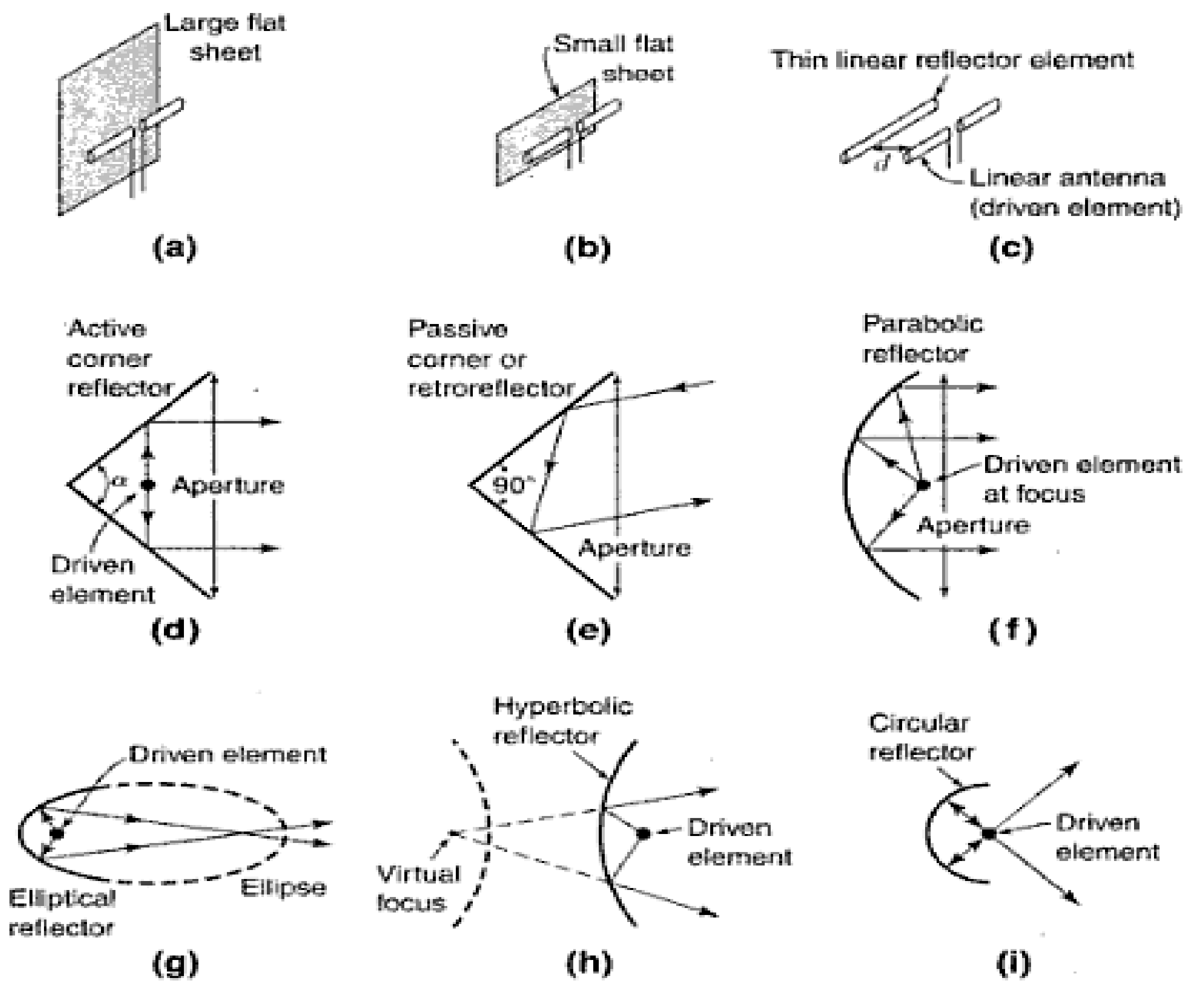
1. Use of substrate with smaller ε_r yields better radiation.
2. Micro strip antennas can have the variation of electric field in both θ and ϕ directions (i.e. azimuthal and elevation angles).
3. The directivity of a patch is given by D =4π/π=4 or 6.021 dbi or in general less than 10 dbi.
4. Gain of a patch is roughly estimated between 7-9 dbi.
5. The width W controls the input impedance and hence the band width of the rectangular strip antenna.
6. Micro strip patch antennas have a very high quality factor. The quality factor Q represents the losses associated with the antenna.
7. The overall efficiency of micro strip antennas will be very poor.
8. A square patch gives circular polarization and a circular patch gives a linear polarized wave.
9. The band width of rectangular patch antennas is typically of the order of 3%.

Impact of different parameters on characteristics: The parameters L, W, h, A and ε_r are Length, Width, Thickness, Area and dielectric constant or permittivity of the dielectric substrate controls the antenna properties. More is the length than width, more is the narrow beam width. The ratio L to W is called as Aspect ratio of the patch which controls the resonant frequency. The permittivity ε_r controls the radiation pattern. Smaller the permittivity better is the radiation. The thickness h has the negligible effect on antenna properties. As a general principle, " An antenna occupying more space in a spherical volume will have a wider band width".

Applications: 1. Increasingly gaining popularity for use in wireless applications due to their low profile structure.

2. Extremely compatible for embedded antennas in hand held wireless devices such as cellular phones and pagers.
3. The telemetry and communication antennas on missiles.
4. Satellite communication, micro wave and milli meter wave systems.
5. Airborne and Spacecraft systems.
6. Defense and Space applications.

Reflector antennas: Reflectors are widely used to modify the radiation pattern of a radiating element. Especially to eliminate the radiation in back ward direction (minor lobes). Several reflector types are illustrated in below figures from (a) to (i).

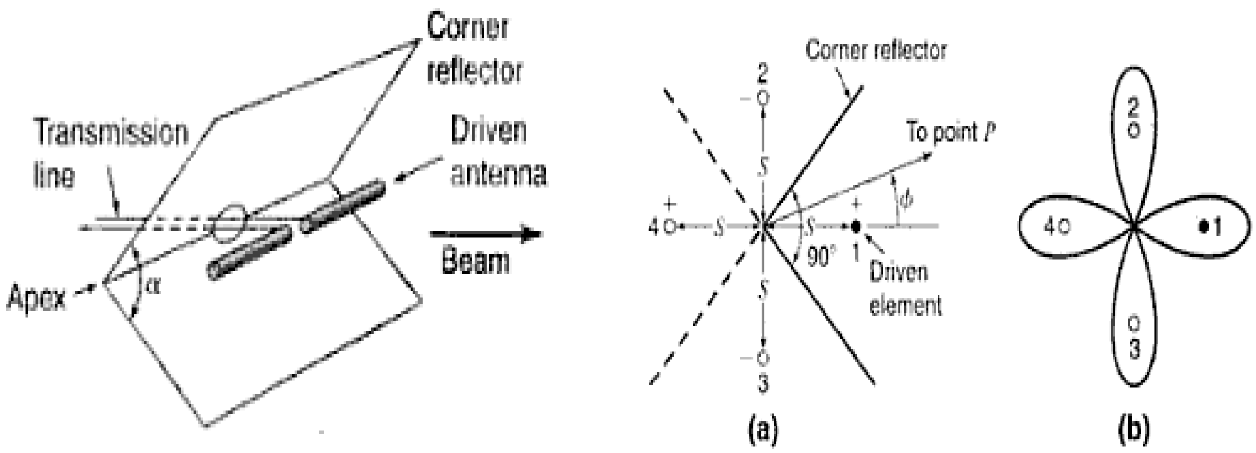


Reflectors of various shapes.

With two flat sheets intersecting at an angle α (<180^0) as in above figure (d), a sharper radiation pattern as from a flat sheet reflector (α =180^0) can be obtained. This arrangement called an active corner reflector antenna is the more practical antenna. A corner reflector without an exciting antenna can be used as a passive reflector antenna. For square corner reflector antennas the corner angle is 90^0.

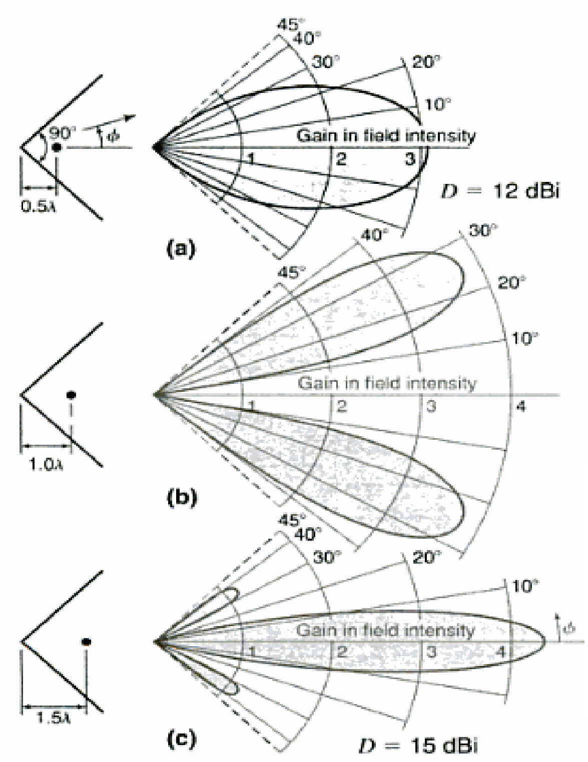
For the analysis of corner reflector antennas the “Method of Images” can be used in which a perfectly conducting plane sheet reflector of infinite extent is assumed to exist.

Corner reflector antennas: Two flat reflecting sheets intersecting at an angle or corner as in below figure form an effective directional antenna. When the corner angle α=90^0, the sheets intersect at right angles, forming a square corner reflector. A corner reflector with α

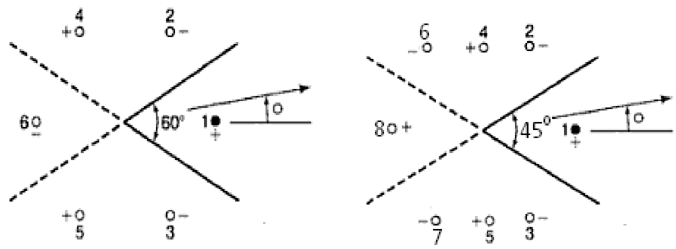


=180^0 is equivalent to a flat sheet reflector and is considered as the limiting case of corner reflectors. Assuming perfectly conducting reflecting sheets of infinite extent, the method of images can be applied to analyze the corner reflector antenna for angles α =180^0/n. Where n is any positive integer. Corner angles of 180^0,90^0,60^0 etc…(for n=1,2,3,…) can be treated in this way. Corner reflectors of intermediate angles cannot be determined by this method but can be interpolated appropriately from the others. In the analysis of the 90^0 corner reflectors there are 3 image elements 2,3 and 4 located as shown in above figure (a). The driven antenna 1 and the 3 images have the currents of equal magnitude. The phase of the currents in 1 and 4 is the same. The phase of the currents in 2 and 3 is the same but 180^0 out of phase with respect to the currents in 1 and 4.

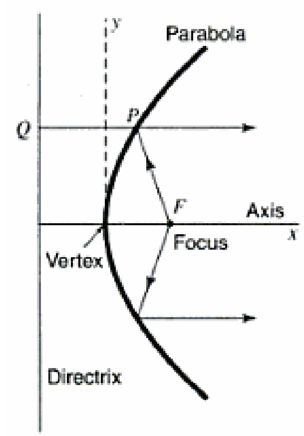
The calculated radiation pattern of a 90^0 corner reflector with an antenna to corner spacing of s=0.5λ is shown in figure (a)If s exceeds a certain value, a multi lobe pattern may be obtained. For example a square corner reflector with s=1.0 λ has a 2-lobed pattern as in figure (b). If the spacing is increased to s= 1.5λ, the pattern is as shown in figure (c) which is obtained with the major lobe in $\phi=0^0$ direction but with minor lobes present.



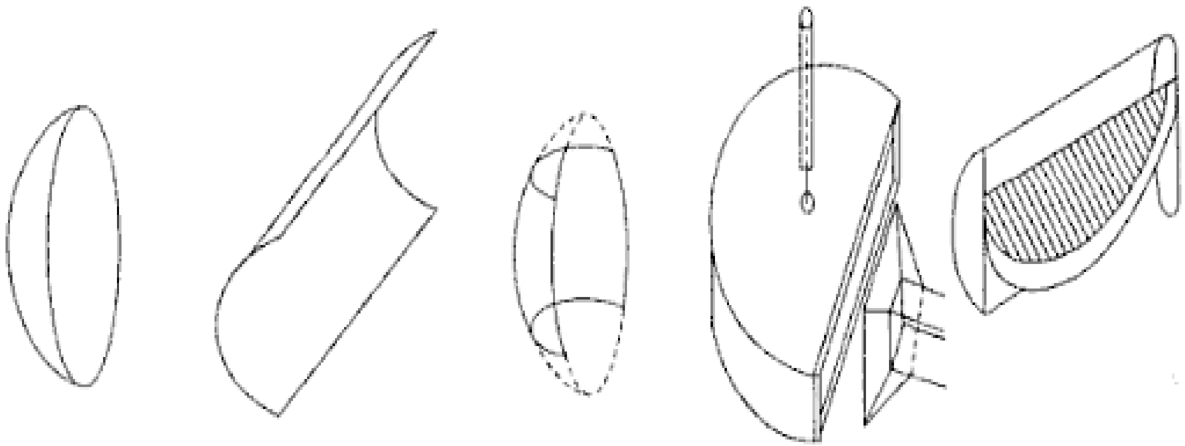
Radiation patterns can be calculated for corner reflectors for angles of 60^0,45^0, etc… in the similar manner. For the 60^0 corner the analysis requires a total of 6 elements 1 actual antenna 5 image elements as shown in below figure (a). And 1 actual antenna and 7 images (total of 8 elements) for 45^0 corner reflector as shown in below figure (b).



The parabolic reflector: The parabola is defined as the locus of the distance from any point p on parabola to a fixed point F called the focus is equal to the perpendicular distance to a fixed line called the directrix. i.e. PF=PQ in the figure. The property of a parabolic reflector is that all the waves from an isotropic source at the focus that are reflected from the parabola arrive at a line AA' with equal phase. The "image" of the focus is the directrix and the reflected field along the line AA' appears as through it originated at the directrix as a plane wave. (i.e. PF + PS=QS). That is parabolic reflector can be used to convert any wave front into a plane wave front which results in a narrow pencil beam type of radiation by satisfying the Fermat's principle (Equality of electrical or optical path lengths). Such a developed parabolic reflector antennas are having a wide range of applications in the present days in the field of communication.



Types of parabolic Reflectors: Different formations of parabolic reflectors can be obtained by rotating a parabola which is a 2-dimensional curve about different ways. The generally used 5 types of parabolic reflectors are shown in below figures from (a) to (e).



(a) (b) (c) (d) (e)

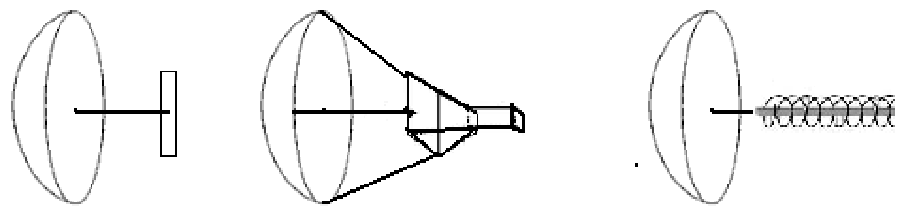
Figure (a) represents a conventional and most commonly used paraboloidal reflector. It is a 3-dimensional curved surface generated by rotating a parabola about its own axis. Figure (b) represents a parabolic cylinder which is generated by moving the parabolic curve normal to its axis. It provides a rectangular mouth and has a line instead of a point as the focus. Figure (c) represents a papered paraboloid in which the edges of the paraboloid are tapered (smoothened). Figure (d) represents the pill box type reflector which can be obtained by shortening the cylindrical parabola. And figure (e) represents a cheese parabola which can be considered as a slice of paraboloid or cylindrical parabola.

Out of all these 5 types the paraboloid or micro wave dish antenna or simply dish antenna was the most popularly used antenna in many Present day applications in wireless communication systems.

Feed systems: The antenna placed at the focus of a paraboloid is known as primary or feed radiator and the parabolic reflector is known as secondary radiator. That is the entire parabolic reflector antenna consists of two basic components the reflector and a source of primary radiation at the focus.

An ideal feed would be that radiator which radiates towards reflector in such a way that it illuminates the entire surface of the reflector and no or zero energy is radiated in other directions. Of course such an ideal radiator is not available in practice. However there are a number of choices for primary feed.

A dipole antenna is not very much suitable for the feed but occasionally used. A most common feed radiator for paraboloid reflector antenna is a wave guide horn. As shown below a horn antenna is pointing the paraboloid in such a way that the entire radiation is falling only on to the parabolic region. Further if circular polarization is required then conical horn or helical antennas can be used as feed at the focus of the paraboloid as shown in below figure (c).



(a) (b) (c)

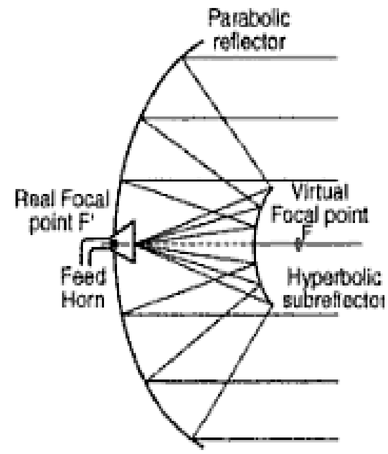
Spillover effect: If the feed antenna used is not perfectly radiating the entire radiation on to aperture or surface of the parabolic region, then the resulted radiation pattern from the parabolic reflector along its axis is not strong some of the energy or radiation will lost. This is called spillover effect.

If the feed or primary antenna is isotropic then the paraboloid will produce a beam of radiation. Assuming the circular aperture is large, the beam width between first nulls is given by FNBW = $\frac{140\lambda}{d}$ degrees. Where λ is the free space wave length and d is the diameter of the aperture. Then the half-power beam width is given by HPBW =$\frac{58\lambda}{D}$ degrees. Finally the directivity, D of large uniformly λ^2illuminated aperture is D =$\frac{4\pi Ae}{\lambda 2}$ =$\frac{4\pi}{\lambda 2}\frac{\pi d2}{4}$ =π^2 $(d/\lambda)^2$ = $9.87(d/\lambda)^2$.

Cassegrain feed: In 18th century Astronomer Cassegrain introduced one feed arrangement. In which the primary feed radiator is positioned around an opening near the vertex of the paraboloid instead of at the focus. This system employs a hyperboloid secondary reflector whose one of the foci coincides with the focus of the paraboloid. The feed radiator is aimed at the secondary hyperboloid reflector or sub reflector. As such the radiation emitted from the feed radiator is reflected from the Cassegrain secondary reflector which illuminates the main paraboloid reflector as if they had originated from the focus. Then paraboloid reflector reflects the rays or waves as usual as shown in figure.

Advantages of Cassegrain feed:

1. To minimize length of transmission line or wave guide connecting the radiator and transmitter.
2. Reduction in spillover effect.
3. Ability to place the feed in at a convenient place.



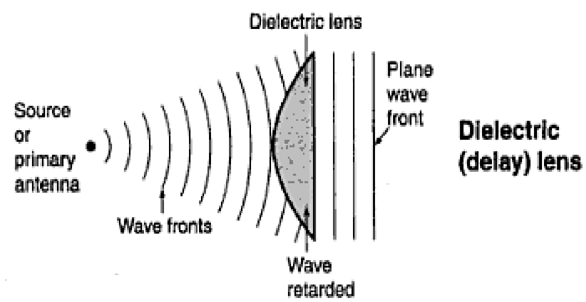
Disadvantage: It is that some of the radiation from the parabolic reflector is obstructed by the hyperbolic region. This can be neglected for greater dimensions of paraboloids. But, it becomes a problem with small dimensional paraboloid.

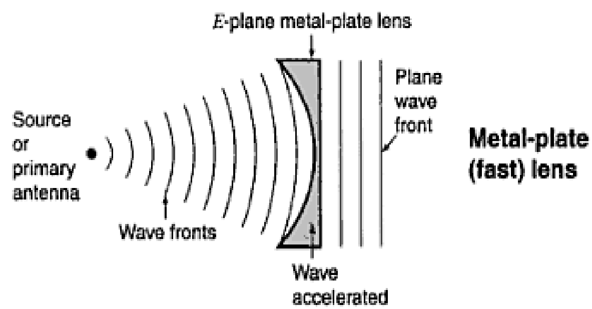
Comparison between parabolic and corner reflectors:

1. It is essential that a parabolic reflector need a directional feed which radiates all or most of the energy into the parabola. A corner reflector on the other hand does not require a directional feed.
2. Parabolic reflector has specific focal point whereas the corner reflector has no focal point.
3. Parabolic reflectors use the Fermat's principle which is nothing but equality of electrical or optical path lengths to give plane wave front. Whereas the corner reflectors will use the Method of images.

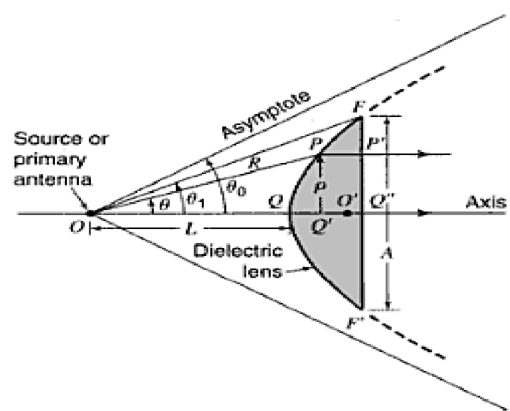
UNIT-6 LENS ANTENNAS

Introduction: Lens antennas may be divided into two distinct types: (1) Delay lens Antennas in which the electrical path length is increased by the lens medium and (2) Fast lens antennas in which the electrical path length is decreased by the lens medium. In delay lens the wave is retarded by the lens medium. Dielectric lenses and H-Plane metal plate lenses are of the delay type. E-Plane metal plate lens are of the fast type. Dielectric lenses may be divided into two groups they are (1) Lens constructed of non-metallic dielectrics such as Lucite or Polystyrene. (2) Lenses constructed of metallic or artificial dielectrics.





Non-Metallic Dielectric Lens Antennas-Fermat's Principle: This type is similar to the optical lens. It may be designed by the ray analysis methods of geometrical optics. If any lens medium is satisfying the Fermat's principle which is nothing but the equality of electrical path lengths of electrical or optical means, will give a plane wave front after the lens medium. For example in the figure shown the electrical path length of the path OPP' must be equal to the electrical length of the path OQQ'Q'' or OP must equal OQ'. Let OQ=L and OP=R and let the medium surrounding the lens be air. From figure



$$\text{Cos}\theta = \frac{L+QQ'}{OP} = \frac{L+QQ'}{R} \text{ or } QQ' = R\text{Cos}\theta - L.$$

$$\text{Then } \frac{R}{\lambda_0} = \frac{L}{\lambda_0} + \frac{R\text{Cos}\theta - L}{\lambda_d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Where λ_0 is wave length in free space and λ_d is the wave length in lens medium. Multiplying The above equation by λ_0, we get $R=L+n(R\text{Cos}\theta L)$……………………………………………………(2).

Where $n= \frac{\lambda_0}{\lambda_d}$ is the refractive index of lens. In general $n=\frac{\lambda_0}{\lambda_d} = \frac{f\lambda_0}{f\lambda_d} = \frac{v_0}{v_d} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_o}}$. Where f is the frequency, v_0 is velocity in free space, μ is permeability of dielectric medium, ε is permittivity of dielectric medium μ_0 is permeability of free space and ε_0 is the permittivity of free space and $\mu = \mu_0 \mu_r$ & $\varepsilon = \varepsilon_0 \varepsilon_r$ Thus finally $n=\sqrt{\mu_r \varepsilon_r}$. For non magnetic materials μ_r is very close to unity so that $n=\sqrt{\varepsilon_r}$. Returning to the above equation (2),

i.e. R =L+n(R Cosθ-L)

= L+nRCosθ-nL

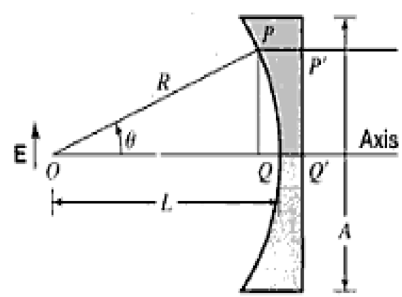
Or R-nRCosθ =L-nL

or R(1-nCosθ) =L(1-n)

or

This equation is called the design equation of lens antenna which gives the required shape of lens.

E-Plane metal plate lens antennas: This type is similar to the optical Plano concave lens. It may be designed by the ray analysis method of geometrical optics. Any Plano concave lens can be used an antenna, if it satisfies the Fermat's principle which says that a plane wave front can be obtained from the lens if the electrical path lengths travelled by the wave or ray are equal. For example, In the figure shown the electrical path length of the path OPP'P" must equal to the electrical length OQQ'Q" or OP' must equal OQ'. Let OQ' =L and OP =R. and let the medium surrounding the lens be air or vacuum. Then from the figure OQ = L-RCosθ =PP' or we can write $\frac{L}{\lambda_0} = \frac{R}{\lambda_0} + \frac{L - R\cos\theta}{\lambda_g}$. Where λ_0 is wave length in free space and λ_g is wave length in lens medium. Then we can write the above equation as



$L = R + \frac{\lambda_0}{\lambda_g}(L - R\cos\theta)$

$L = R + n(L - R\cos\theta)$ $\left(\frac{\lambda_0}{\lambda_g} = n\right)$

Or R-nRCosθ =L-nL

or R(1-nCosθ) =L(1-n) or

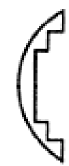
This equation is called the design equation of lens antenna which gives the required shape of lens.

H-Plane metal plate lens antennas: In this case the refractive index is equal or greater than unity. H-Plane metal plate lens can be designed similar to the E-Plane metal plate lens by applying the principle of equality of electrical path lengths i.e. satisfying the Fermat's Principle. This type of lens called an H-Plane type since the plates are parallel to magnetic field or perpendicular to E-Plane. For this case also the design equation is given by

$$R = \frac{L(1-n)}{(1-n\cos\theta)} = \frac{L(n-1)}{(n\cos\theta - 1)}$$

Tolerances on Lens antennas: In a dielectric lens, differences in path length may be caused by deviations in thickness from original thickness and also by the variations in the refractive index of the lens material used. Assuming an allowable variation of $\lambda_0/32$, we have the thickness tolerance that $\frac{\Delta t}{\Delta d} = \frac{\Delta t}{\lambda_0} = \frac{1}{32}$ or $\Delta t = \frac{\lambda_0}{32(n-1)} = \frac{\lambda_d}{32(n-1)} = \frac{0.03\lambda_0}{n-1}$. Thus we can say that the variation in thickness of the lens shows that there will be a corresponding change in electrical path lengths.

Zoning in lens antennas: In lens antennas the equality of path lengths can be maintained by using a method called zoning of lens to get a plane wave front after the lens medium. The zoning is nothing but the division of the plane face of the lens into different regions as shown in the figure. The no. of zones or divisions will depend upon the dimensions of the lens used. For example in the figure, we are having 5 zones, z_1, z_2,z_3,z_4 and z_5 to have or to maintain equal electrical path lengths through all zones.

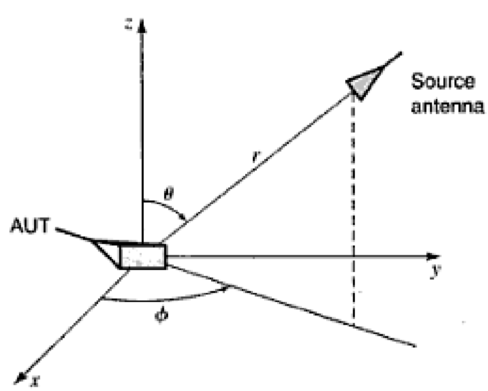


Zoned lens

Applications of lens antennas: Lens antennas can be used in almost all the applications to which reflector antennas can be used. But, lens antennas will be preferred where the size of the antennas required must be small. With this point of view the lens antennas are very suitable on planes. The lens antennas find more applications in millimeter waves where the size of the antennas required is to be small. Finally we conclude that the lens antennas can be used where the size of antenna is required to be compact.

Antenna Measurements: Antenna Measurements are needed to know the actual performance of the designed antennas and also where they have to be used. There are no accurate antenna measurements practically. Ideally or theoretically we can have accurate measurements. These theoretical concepts can be used to estimate or predict the actual performance of antennas in terms of their gain, pattern, polarization, band width and efficiency etc…

Basic Concept: The most common antenna measurement is to measure its radiation properties like directional pattern, gain or phase pattern in the far field. Typical configuration of the measurement of radiation is shown in the figure. The basic procedure is to place transmitting or receiving source antenna at different locations with respect to the Antenna Under Test (AUT) and thus get a no. of samples of the pattern at different locations. All this arrangement will have good accuracy if the medium selected is an anechoic chamber or free space.



Reciprocity in Antenna Measurements: The Antenna Under Test (AUT) can acts as either a receiving or transmitting antenna. This is due to the applicability of reciprocity theorem. However to apply this reciprocity theorem in practical antenna measurements, the conditions to be satisfied are:

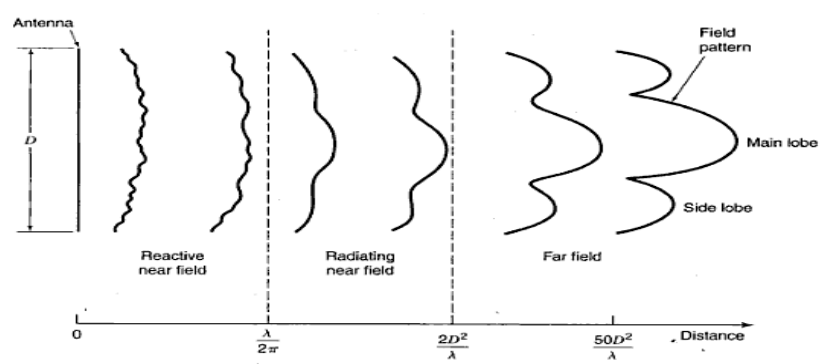
1. The media are linear, passive and isotropic.
2. The emf's in the terminals of the interchanged antennas are of the same frequency.
3. The power flow must be same in either direction.

In standard careful measurements the first condition can be satisfied always and the second can be met without problems. The third condition is perhaps the most difficult and may lead to the statement that there is no accurate antenna measurement.

Near and Far fields: Based on Huygen's principle it does not matter, where the sampling of the field takes place provided enough information is obtained on some surfaces surrounding the AUT. However the measuring practices and constraints depend largely on the distance of the surface from the AUT. It has been defined that one can find several regions of the radiated field in the vicinity of the antenna. These are the reactive near field region, the radiative near field or Fresnel region and the far fields or Fraunhofer region. As we are almost always interested in the radiation properties in the far field, it is obvious that the measurements usually also takes place in the far field. There are several advantages of the far field measurement they are:

1. The measured field pattern is valid for any distance in the far field region only simple transformation of the field strength according to $\frac{1}{r}$ is required.
2. If the power pattern is required, only power measurement is needed.
3. The result is not very sensitive for changes in the location of the phase center of the antennas and thus the rotation of the AUT does not cause significant measurement errors.
4. Coupling and multiple reflections between the antennas are not significant.

The main disadvantage of far field measurement is the required large distance between antennas leading to large antenna ranges. However because of the large no. of advantages most of the cases the measurement will be done in the far fields. The below figure shows the radiation patterns in the near and far field regions.



Radiation patterns in near-field and far-field regions.

Coordinate System: The important parameters of the antenna measurements are radiation properties. Since the radiation patterns of the antennas are having 3-dimensional variations in the space, we can consider the spherical coordinate system for the measurements of antennas. In IEEE standard spherical coordinate system θ is the elevation angle measured from z-axis and ϕ is azimuthal angle measured from x-axis and r is the radius of the sphere.

Sources of errors in antenna measurements: Any measured quantity has a margin of error. Thus, the complete value for the gain of an antenna might be 15dBi $\pm$ 0.5dBi indicating a half decibel uncertainty. To reduce the measurement uncertainty to an acceptable level, the critical sources of errors identified in antenna measurements are given below

1. Amplitude variations.
2. Phase variations.
3. Coupling to the medium.
4. Alignment errors.
5. Instrumental errors.
6. Leaking in cables.
7. Man made signals.
8. Atmospheric effects.
9. Impedance mismatches.

Measurement of different antenna parameters: To describe an antenna completely, several radiation and circuit parameters are needed.

Pattern Measurement: The directional pattern, power or amplitude pattern is usually measured by rotating the AUT on a positioned and by detecting the received power from the power meter attached to it in the basic antenna measurement arrangement or concept. Then plotting this power verses angle we get the pattern. Beam width pattern shape, side lobe levels and their directions and null directions are parameters which can be obtained easily from the measured pattern.

Directivity Measurement: The directivity, D of an antenna can not be measured directly from the pattern measurement of antennas. However it can be computed from the normalized power pattern Pn(θ,ϕ) as $D = \frac{4\pi}{\iint Pn(\theta,\varphi) Sin\ \theta\ d\theta\ d\varphi}$. That is the normalized power must be first calculated from the different power readings from the meter attached to AUT in different θ and ϕ directions and substituting this normalized power in the above formula we can measure the directivity of antenna.

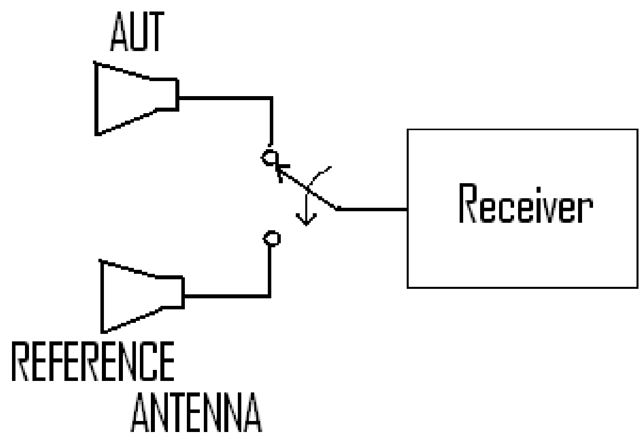
Gain Measurement: There are different methods which can be used to measure the gain of the antenna. Absolute method and comparison method are the two basic gain measurement methods.

Absolute Method: The absolute method is based on the Frii's transmission formula $P_R=P_T G_T G_R(\frac{\lambda}{4\pi R})^2$ watts. Where P_R is the power received, P_T is the power transmitted, G_T is gain of transmitting antenna G_R is gain of receiving antenna, λ is wave length and R is the distance between the antennas. In this absolute method we will assume that G_T and G_R are same as that of AUT used in basic arrangement. Then by knowing the values of λ and R we can measure the power received. Then the ratio of the powers $\frac{P_R}{P_T}$ will give the power gain of that antenna.

The efficiency of the measurement can be improved by increasing the no. of antennas in this method. In two antennas method, two identical antennas are required (G_T = G_R). If two antennas of identical gains are not available, then a third antenna is needed. In 3-antennas method three sets of measurements are performed, one with each pair of antennas. From these three simultaneous equations, the three unknown gains can be solved in straight forward method.

There are many sources of errors in the absolute method. The misalignment of antennas, polarization mismatch, and impedance mismatch both in the transmitting and receiving sections. However the efficiency can be still increased by increasing the no. of antennas more than 3 also. But, we have to compromise for the misalignments of antennas used.

Comparison Method: The comparison method can be called as gain-transfer method. In this method, the power received with the AUT and with a unknown reference antenna are compared as shown in figure. This measurement gives $G_{ref} = \frac{P_{AUT}}{P_{ref}} G_{AUT}$. Where P_{AUT} is power received with AUT, P_{ref} is power received with reference antenna, G_{ref} is gain of reference antenna and G_{AUT} is gain of AUT.



In this method the unknown gain G_{ref} can be determined by knowing all other parameters. For each measurement we compare the result and we can select the average value as the gain of the reference or unknown antenna. This measured gain can be applicable to reference antennas of any polarization.

The gains of circularly or elliptically polarized antennas are usually determined by measuring the partial gains for two orthogonal linear polarizations i.e. Horizontal and Vertical polarizations. That is the total gain G_{AUT} is the sum of two partial gains as $G_{ref} = G_H + G_V$. Where G_H is gain of reference antenna at horizontal polarization and G_V is gain of reference antenna at vertical polarization.

UNIT-7 Wave Propagation-I

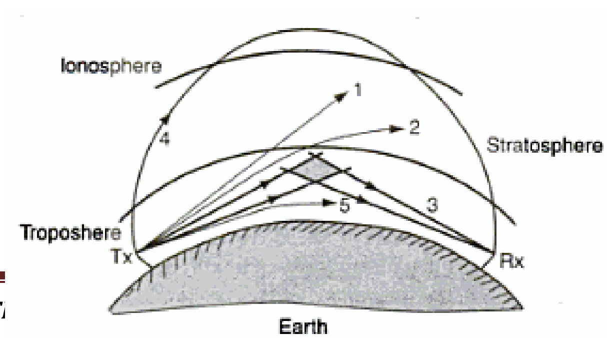
Introduction: The energy generated by a transmitter is fed to transmitting antenna which in turn radiates the same into the space. This radiated energy travels all through the space and this mode of travel is termed as wave propagation. The space between the transmitting and receiving antennas is nothing but the atmosphere present above the earth. This atmosphere can be divided for the analysis of study as Troposphere, Stratosphere and Ionosphere.

Troposphere: It extends from the earth's surface to 10 t0 15kms. The temperature of troposphere decreases with height at an average rate of 6 degree/km. There may be a lot of turbulence because of variations in temperature, pressure and density. The upper boundary of troposphere is called tropopause.

Stratosphere: It extends from 15 t0 40kms above the ground surface. The temperature throughout this region is almost constant and there is a little water vapor present. The stratosphere has almost no effect on radio waves.

Ionosphere: It extends from 40 t0 400kms above the ground surface. The temperature in this region will increases as the height increases. The changes in the ionosphere due to ionization of the particles are very turbulent.

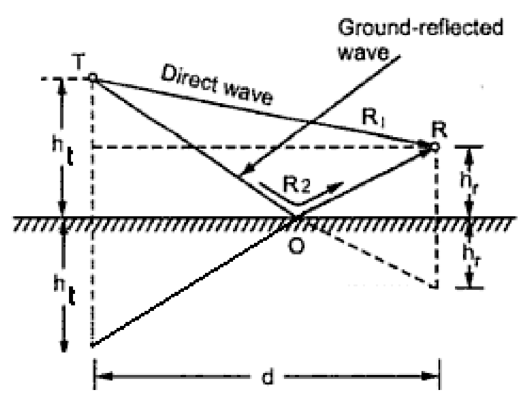
Modes of wave propagation: The energy radiated from the transmitting antenna may travel all through space with or without alteration in its characteristics depending on the properties of the space between transmitting and receiving antennas. Generally in free space or space the wave may undergo either reflection or refraction or diffraction or scattering under normal variations taking place in the free space. Due to all these phenomena. Finally the wave can reach the receiving antenna due any one of the following modes of propagations.



The wave marked1 adopts a straight line path in free space and is sometimes referred as direct wave (DW). The wave marked2 adopts a curved path in view of refraction phenomena in the atmosphere. The wave marked3 reflected or scattered in the troposphere is termed as tropospheric wave. This mode of propagation is the result of irregularities of troposphere, which extends to nearly 10 t0 15kms from the earth's surface. The communication utilizing the tropospheric waves is called tropospheric propagation. The wave marked4 reaching the receiver after getting refracted and reflected from the ionosphere is called ionospheric or sky wave. This is sometimes also referred as ionospherically reflected or ionospherically scattered wave. The wave marked5 propagating over paths near the earth's surface is termed as ground wave or surface wave. The ground waves are vertically polarized and exist if antennas are closer to earth. Out of all these modes of propagations the generally occurring propagations are only three as (1)Ground or surface wave (2) Space or tropospheric wave (3) Ionospheric or sky wave propagation.

Ground wave propagation: The energy propagated over paths near the earth's surface is considered to be made possible through ground waves. The earth's surface is normally considered to be flat or plane , provided the distance between the transmitter and receiver does not cross a barrier d which is given by d = $\frac{50}{f^{1/3}}$ in miles. Beyond this distance the effect of curvature of earth is to be accounted. A ground or surface wave can exist if the polarization of the wave is vertical at transmitter and antennas are situated closer to earth. For these types of waves the ground can be considered as perfectly reflecting plane.

Plane earth reflection: For elevated transmitting and receiving antennas within in the line of sight of each other, the received resultant signal is a combination of the signal reaching the receiver through a direct path and that reaching after being reflected by the ground. The flat or plane earth can be of two types as smooth and rough surface based on Rayleigh criterion given by R = $\frac{4\pi\sigma Sin\theta}{\lambda}$. Where σ the standard deviation of the surface irregularities relative to the mean surface height, θ is the angle of incidence, λ is the wave length. If R<0.1, The reflecting surface is considered as smooth surface. If R>10, the reflecting surface is considered to be rough.



Field strength Relation: In the figure, the transmitting antenna, T located at A, with height ht and receiving antenna R is located at B with height hr. R1 is the distance travelled by Direct wave (DW), R2 is the distance travelled by the reflected wave (RW) by earth and image of transmitting antenna since the earth is assumed to be flat and perfectly conducting. From the upper right angle triangle of the figure

$R_1^2=(h_t - h_r)^2 + d^2$ or $R_1=d\,[1+(h_t - h_r)^2/d^2]^{1/2}$

Similarly from the lower right angle triangle, $R_2^2 =(h_t + h_r)^2 + d^2$

$$R_2^2 = (h_t + h_r)^2 + d^2 \text{ or } R_2^2=d^2[\frac{(h_t+h_r)^2}{d^2} +1]$$

$$R2 = d\sqrt{\frac{(h_t+h_r)^2}{d^2}} +1$$

But, generally, the distance d will be much greater than the heights of antennas h_t and h_r. Then we can write the above equation as

$R_1=d\,[1+(h_t - h_r)^2/2d^2]$

$R_1=d+(h_t - h_r)^2/2d$ and $R_2=d+(h_t + h_r)^2/2d$

The difference in path lengths R_2-R_1 is given by

$$R_2-R_1 =\frac{(h_t+h_r)^2-(h_t-h_r)^2}{2d} = \frac{2h_th_r}{d}$$

The phase difference corresponding to this path difference is

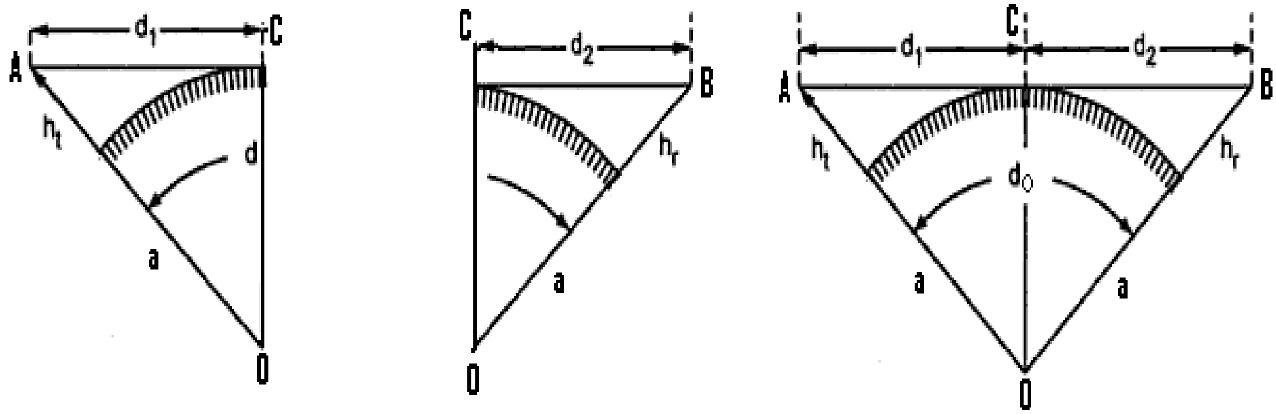
$(\frac{2\pi}{\lambda})\ (\frac{2h_th_r}{d})\ =\frac{4\pi h_th_r}{\lambda d}$ If E_0 is the field produced at unit distance by direct ray from transmitting antenna, then the total electric field is $E =(\frac{E0}{d})\ (\frac{4\pi h_th_r}{\lambda d})\ =\frac{4\pi E0 h_th_r}{\lambda d2}$

$$\text{Or } E = E_0\,\frac{4\pi h_th_r}{\lambda d^2}.$$

According to Sommerfeld there is no difference between Ground and space waves.

Curved Earth Reflections: The curvature of the earth can be neglected if the distance is lesser than $d=\frac{50}{f^{1/3}}$ when this distance increases, then the losses due to earth will be also increases and finally reduces the original wave energy. To analyze the effect of the curved earth on wave propagations, we need the horizon of the earth.

Horizon of the Earth:



In the above figures 'a' is the radius of the earth, h_t is the height of the transmitting antenna, h_r is the height of the receiving antenna, d_1 is the curvature of the earth at transmitter side and d_2 is the curvature of the earth at receiver side and do is the total curvature or horizon of the earth from transmitter to receiving antennas α is the angle between two lines of radius of the earth and transmitting or receiving antennas. From figure (a) we can write Cos α $=\frac{a}{a+h_t} \cong 1-\frac{h_t}{a}$……………………... (1)

In practical problems α is very small. Thus for small α, we can write

Cos α $=1-\frac{\alpha^2}{2}$……………………………… (2)

From equations 1 and 2 $1-\frac{\alpha^2}{2} = 1-\frac{h_t}{a}$

$\frac{\alpha^2}{2} = \frac{h_t}{a}$ Or $\alpha = \sqrt{\frac{2h_t}{a}}$ One more approximation for α when it is small is given by $\alpha = \frac{d_1}{a}$………………………..(3)

Therefore $\frac{d_1}{a} = \sqrt{\frac{2h_t}{a}}$. Thus the horizontal distance $d_1=\sqrt{2ah_t}$ meters.

Similarly, from figure (b), we get $d_2=\sqrt{2ah_r}$ meters.

If figures (a) and (b) are joined together by overlapping OC, it gives figure (c) then the total distance or horizon d_0 can be given by $d_0 = d1 + d2 = \sqrt{2a\ (\sqrt{h_t}\ + \sqrt{h_r})}$

$d_0 = \sqrt{(2X6.37X10^6(\sqrt{(h_t)}\ (m) + \sqrt{h_r}(m))}$ [since a=6.37X10^6 meters]

$d_0 = 3.57\ \sqrt{h_t}(m) + \sqrt{h_r}(m))$ Km.

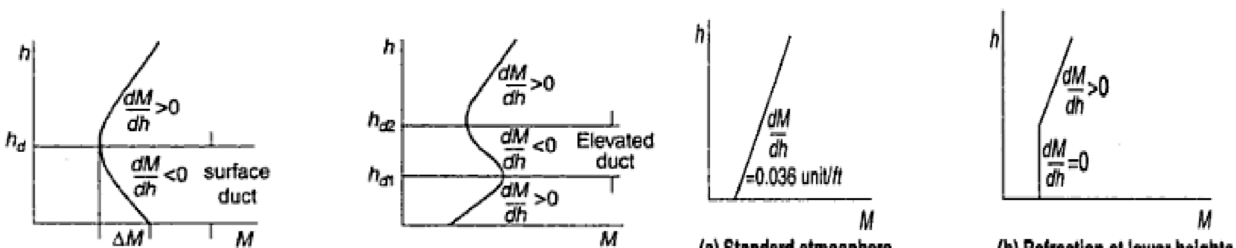
The horizon d0 can be termed as Line-of-sight (LOS) distance.

Wave tilt: It is defined as the change of orientation of the vertically polarized ground wave at the surface of the earth. During the passage of travel, the wave starts tilting in the forward direction as it progresses. The magnitude of tilt will depend on the conductivity and permittivity of the earth.

Tropospheric Scattering: Reception of the signals very far beyond the horizon in VHF and UHF range is possible due to scattering phenomena in the troposphere. The troposphere is turbulent at these frequencies. Waves passing through such turbulent regions get scattered. When λ of the wave is large, then this scattering takes place in all the directions. To receive this scattered signal at a point well beyond the horizon, the transmitting and receiving antennas must be so oriented that their beams of radiation overlap in a region where forward scattering is taking place. This scattering process is random of nature. Generally signals of above 500 MHz will be reached to receiving antenna using these scattering phenomena.

Power received in free space: Consider an isotropic or Omni directional antenna which radiates uniformly in all directions. Let P_t be the transmitted power, P_r be the received power in free space at a distance, R. Then P_r is the transmitted power divided by the surface area $4\Pi R^2$ of an imaginary sphere of radius R. Thus, $P_r = \frac{P_t}{4\pi R^2}$ watts. If the transmitting antenna is directional with gain G_t then $P_r = \frac{P_t G_t}{4\pi R^2}$ watts. If the effective aperture of the receiving antenna is A_r then $P_r = \frac{P_t G_t A_r}{4\pi R^2}$ watts. If the gain of the receiving antenna $G_r = \frac{4\pi A_r}{\lambda^2}$ then substituting $A_r = \frac{\lambda^2 G_r}{4\pi}$ we get $P_r = \frac{P_t G_t G_r \lambda^2}{4\pi R^2}$ watts.

Super Refraction, M-Curves and Duct Propagation: Other than surface and space wave propagations, depending on the changes taking place in the troposphere refractive index, n there may be some rare cases of wave propagations like super refraction and duct propagations. That is the variation of n leads to the phenomenon's such as super refraction, fading and ducting. The duct can be assumed to be a wave guide with leakage. In such



(c) surface duct formed over sea (d) elevated duct formed over land (a) Standard atmosphere (b) Refraction at lower heights equals earth's curvature

cases the actual 'n' will be replaced by modified index 'M' as M=n+$\frac{h}{a}$ where h is the height of the antenna and a is the radius of the earth (a=6037x10^6 meters). This modified refractive index 'M' will with respect to the height. The different cases for different values of $\frac{dM}{dh}$ called M-Curves are shown below.

As shown in the above figures, the figure (a0 gives the variation of M with respect to h which is linear. This case happens for standard atmosphere. Figure (b) shows the refraction at lower heights equals earth's curvature. Figure (c) shows the surface duct formed over sea. Figure (d) shows elevated duct formed over land.

Fading and Path loss: The troposphere signals often suffer from fading which is a phenomenon of reduction of signals due to variation in refractive index. This phenomenon occurs when there are sudden changes in the temperature, pressure and humidity of air. The fading can be fast or slow or single path or multi path. To avoid these we can use the methods like space diversity, frequency diversity, time diversity, modulation diversity and polarization diversity.

In addition to the ground effect, reflection, refraction, scattering, diffraction, super refraction, duct and fading effects there may be losses that occur for the waves travelling in free space due to the presence of rain, snow, fog, clouds, birds, trees, mountains and hills etc… Due to all these effects it is necessary to estimate the total path losses occur for the wave travelling in free space. The basic path loss for general communication is given by the relation

$$\text{Path loss} = 32.45 + 20 \log_{10} f \text{ (MHz)} + 20 \log_{10} d \text{ (km)}$$

The total path loss in dB's= $L_{total} = L_{fs} + L_s + L_{ref} + L_{fad} + L_{cpl} + G_t + G_r$

Where L_{fs} is free space path loss, Ls is scattering loss, L_{ref} is refractive index loss, L_{fad} is fading loss, L_{cpl} is coupling loss, G_t is gain of the transmitting antenna and G_r is the gain of the receiving antenna.

Problem: (1) Find the basic path loss for communication between two points 3000kms apart at a frequency of 3GHz?

Solution: Given: f = 3GHz

d = 3000Kms

Required: path loss =?

We know that Path loss = $32.45 + 20\log_{10}f\,(MHz) + 20\log_{10}d\,(km)$

For the given values of d and f, path loss = $32.45 + 20\log_{10}3000 + 20\log_{10}3000$

Path loss =32.45 =20x3.4771+20x3.4771=171.534 dB's

Path loss=171.534 dB's

Problem: (2) Calculate the maximum distance at which signal from transmitting antenna with 144m height would be received by the receiving antenna of 25m height?

Solution: Given: h_t= 144m

h_r = 25m

Required: d_0 =?

We know that $d_0 = 3.57(\sqrt{h_t} + \sqrt{h_r})$km

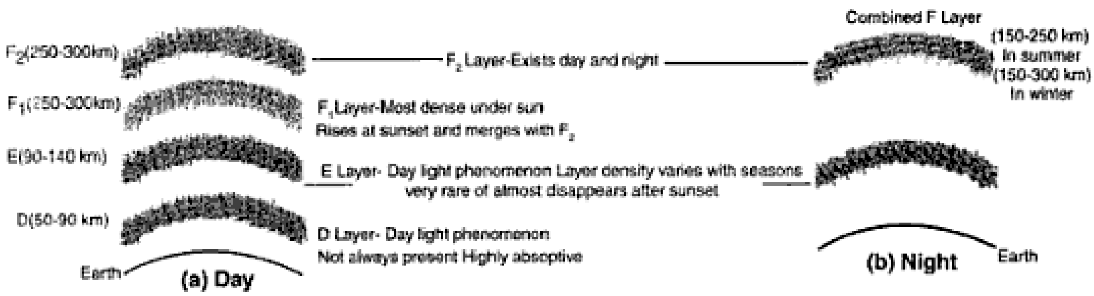
For the given values of h_t and h_r, d_0= 3.57 (12 + 5)Km

d_0=60.69km

UNIT-8 Wave Propagation-II

Introduction: The propagation of sky waves also called as ionospheric waves revolves around the refraction mechanism in the ionosphere. Therefore in order to see the wave propagation in the ionospheric region, it is required to see the structure of ionosphere.

Structure of ionosphere: The ionosphere is a region above the earth and is composed of ionized layers. In general 4 layers namely D, E, F1 and F2 as shown in the figure.



D-Layer: It exists between 40 t0 90 kms above the Earth's surface. It is a day-time phenomena and largely absent in the night. Ionization in the D-Layer is low because less ultraviolet light penetrates to this level. At LF and MF ranges this layer is highly absorptive and limits day-time communication to about 300kms. It is responsible for much of day time attenuation.

E-Layer: It exists between 90 to 140 kms above the Earth's surface. The ionization is almost constant in this layer with little diurnal or seasonal variations. It is closely governed by the amount of ultraviolet light from the sun and uniformly decays with the time at night. The layer permits medium distance communication in LF and HF bands.

F1-Layer: It exists between 140 to 250 kms above the Earth's surface. This layer is almost constant with little diurnal or seasonal variations.

F2-Layer: It exists between 250 to 400 kms above the Earth's surface. At night F1 layer slightly rises and F2 layer slightly lowers to form one layer which is again called the F2 layer. It is some times also referred to as F-Layer. It is more variable in nature. The F2 or F layer is responsible for the most of the HF long distance communication.

Sporadic E-Layer: This layer is a result of an anomalous phenomenon and falls under the category of irregular variations. Its occurrence is quite unpredictable and is observed both during day and night. The cause of its appearance is still uncertain. It occasionally occurs around the E-Layer at discrete locations and then disappears.

Refraction & Reflections of the Sky waves by the Ionosphere: The refraction and reflection phenomenon in the ionosphere is governed by the following three equations.

$n=\sqrt{1-\frac{81N}{f^2}}$.. (1)

$\sin\phi_i=\sin\phi_r$ .... (2)

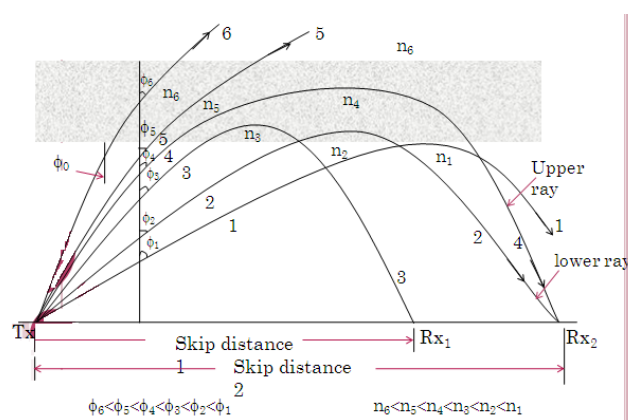
$V_p = c/n$ or $v_g = cn$ or $v_p v_g = c^2$.............. (3)

where n is the refractive index of the ionosphere, N is the number of electrons per cubic centimeter (Electron Density) , f is the frequency in khz, ϕ_r is the angle of refraction, ϕ_i is the angle of incidence, c is the velocity of light, v_p is the phase velocity and v_g is the group velocity. A critical study of above three equations gives the following results:

1. For n>1, $\frac{81N}{f^2}$ term should be negative. Which is not possible thus this condition does not exists.
2. For n<1, $\frac{81N}{f^2}$ should be less than one or unity. This condition always exists. For this condition we can say v_p is always greater than c.
3. For n=1, $\frac{81N}{f^2}$ should be zero. In this case $v_p=c$.
4. For n=0, $\frac{81N}{f^2}$ should be one or unity or $81N=f^2$. In this case $v_p=\infty$.
5. For n negative, $\frac{81N}{f^2}$ should be greater than one or unity or $f^2<81N$, In this case n is an imaginary quantity. In this case ionosphere totally absorbs the wave.

From all the above conditions we can conclude that the wave propagation in ionosphere is totally depends on the refractive index, n of the particular layer of ionosphere.

Ray path: The path followed by a wave or ray is termed as Ray Path. The figure gives 5 different ray paths.



Critical Frequency (f_c): The highest frequency that returns from an ionospheric layer at vertical incidence is called the Critical Frequency, f_c for that particular layer. It is proportional to the square root of maximum electron density in that layer i.e.fc =$\sqrt{81N_{max}}$.

Maximum Usable Frequency: The maximum possible value of frequency at which the wave returns from the ionosphere other than the vertical incidence is called the Maximum Usable Frequency (MUF). The MUF will be always lesser than the Critical Frequency.

Relation between f_c and F_{muf}: If ϕ_i is the incident angle and ϕ_r is the refracted or reflected angle then from Snell's law n = $\frac{Sin\varphi_i}{Sin\varphi_r}$ = $\frac{Sin\varphi_i}{Sin\ 90^0}$ (at f_c, ϕ_r=90^0)

$$n = Sin\varphi_i = \sqrt{1-\frac{81N_{max}}{f_{muf}^2}}$$

$$Sin^2\phi_i = 1-\frac{81N_{max}}{f_{muf}^2}$$

$$Sin^2\phi_i = 1-\frac{f_c^2}{f_{muf}^2} \qquad (f_c = \sqrt{81N_{max}})$$

$$\frac{f_c^2}{f_{muf}^2} = 1- Sin^2\phi_i = Cos^2\phi_i$$

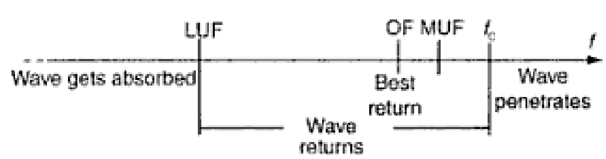
$$f_{muf}^2 = \frac{f_c^2}{Cos_{\varphi i}^2}$$

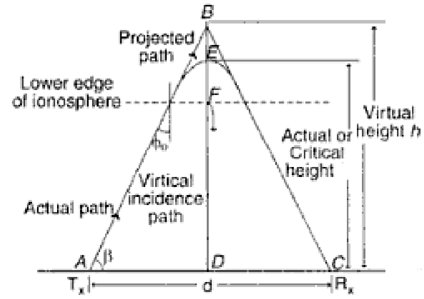
$$f_{muf}^2 = f_c^2 Sec_{\varphi i}^2$$

f_{muf}=f_c Secϕ_i is called Snell's Law.

Lowest Usable Frequency(LUF): The frequency below which the entire power or energy or wave gets absorbed by the ionosphere is referred to as the Lowest Usable Frequency (LUF).

Optimum Frequency (OF): The frequency at which there is optimum return of wave energy is called Optimum Frequency (OF) or sometimes it is also called as the Optimum Working Frequency (OWF). The figure illustrates the LUF, OF, MUF and fc on the frequency scale. The values of all these frequencies are different for different layers.



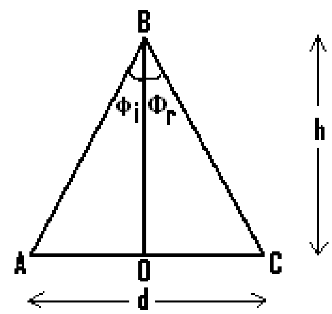


Virtual Height (h): It may be defined as the height to which a wave sent vertically upward and travelling with the speed of light would reach the receiver by taking same two-way travel time. The virtual height, h will be always greater than actual height as shown in the figure.

Skip distance (d): The minimum distance at which the wave returns to the ground at some critical angle is called the skip distance. That means there may be different ray paths for different skip distances between the transmitting and receiving g antennas.

Relation between MUF and Skip distance: From the figure, h is the virtual height, d is the skip distance ϕ_i is the angle of incidence and ϕ_r is the angle of reflection. From the right angle triangle AOB, We get Cos ϕ_i $=\frac{OB}{AB}=\frac{h}{\sqrt{h^2+(d^2/4)}}=\frac{2h}{\sqrt{4h^2+d^2}}$

From secant law we know that $\frac{f_c^2}{f_{Muf}^2}=\cos^2\phi_i$



$$\cos^2\phi_i=\frac{4h^2}{4h^2+d^2}=\frac{f_c^2}{f_{Muf}^2}$$

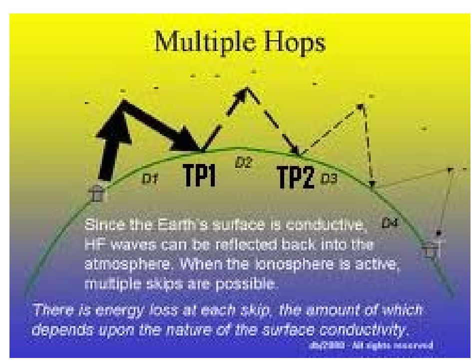
$$\frac{f_c^2}{f_{Muf}^2}=\frac{4h^2+d^2}{4h^2}$$

$$\frac{f_{MUF}}{f_c}=\sqrt{\frac{4h^2+d^2}{4h^2}}=\sqrt{1+\frac{d^2}{4h^2}}$$

$f_{MUF}=f_c\sqrt{1+\frac{d^2}{4h^2}}$ This equation gives MUF interms of skip distance. Otherwise from $\frac{f_{MUF}^2}{f_c^2}=1+\frac{d^2}{4h^2}$ or $(d/2h)^2=\frac{f_{MUF}^2}{f_c^2}-1$ or $d=2h\sqrt{\frac{f_{MUF}^2}{f_c^2}-1}$

This equation gives Skip distance in terms of MUF.

Multiple hop propagation: The wave which will travel from transmitting antenna to receiving antenna without touching the ground and where in between is called a hop distance. A single hop will be equal to the skip distance. For specific applications where the distance between the antennas is very large, more than one hop distance is required. In such cases the wave



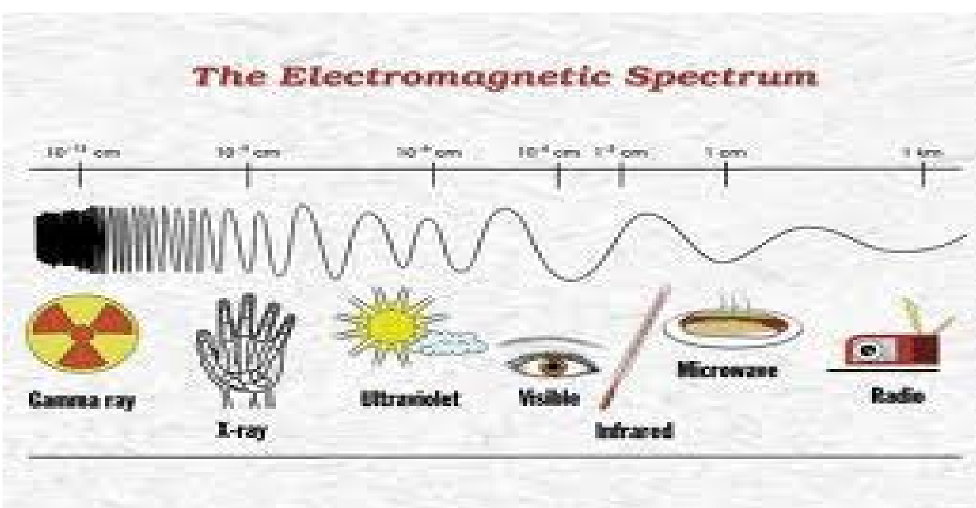
will reach the receiver or target by using multi-hop propagation. In multiple hop propagation, there will be transponders or repeaters at ends of hop distances as shown in the figure. The purpose of these transponders is to again transmit the signal towards the ionosphere with the same characteristics of the wave used at the original transmitter. In the figure TP_1 and TP_2 are the transponders used. d_3 is the skip distance.

Sudden changes in Ionosphere: In addition to all the phenomena studied in the ionospheric wave or sky wave propagation, there are some more phenomena which can takes place in the ionosphere and these will also shows their effect on the sky wave propagation. Such phenomena's are like sunspots, sudden ionospheric disturbances (SID), Ionospheric storms etc…

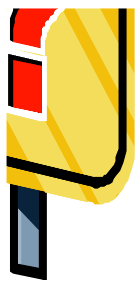
Sunspots: These are the dark irregularly shaped areas on the surface of the sun which keep on appearing and disappearing in two cycles, every 27 days and every 11 years. The occurrence of sunspots, their life span, shape, size and location are all variable and unpredictable. These sunspots cause variations in the ionization level of the ionosphere and hence affect the propagation of characteristics of the waves.

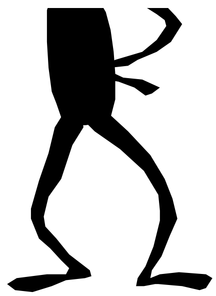
Sudden Ionospheric Disturbances (SID): These may occur any time and may lose from minutes to several hours. The occurrence of SID is due to solar eruptions producing intense bursts of ultraviolet light which are not absorbed by the upper ionospheric layers. These sudden changes will affect the ionospheric wave propagation.

Ionospheric storms: These are caused by disturbances in the earth's magnetic field and are related to solar eruptions and the 27-day cycle of the sun. The effect of these storms may lead to turbulent ionosphere and erratic sky wave propagation. These storms mainly affect the F2 layer reducing its ion density and causing critical frequencies to be lower.



HOW TO PREPARE FOR THE AWP EXAM





HOW TO PREPARE FOR THE AWP EXAM

ACTUAL ORDER OF UNITS
1.ANTENNA FUNDAMENTALS
2.THIN & LINEAR WIRE ANTENNAS
3.ANTENNA ARRAYS
4.VHF, UHF & MW ANTENNAS-I
5.VHF, UHF & MW ANTENNAS-II
6.LENS ANTENNAS
7.WAVE PROPAGATION-I
8.WAVE PROPAGATION-II

PREPARATION ORDER OF UNITS
1. WAVE PROPAGATION-II
2. WAVE PROPAGATION-I
3. LENS ANTENNAS
4. VHF, UHF & MW ANTENNAS-II
5. VHF, UHF & MW ANTENNAS-I
6. ANTENNA FUNDAMENTALS
7. ANTENNA ARRAYS
8. THIN & LINEAR WIRE ANTENNAS

Note : This plan is prepared strictly according to the examination pattern to score maximum marks and not for your own interest.