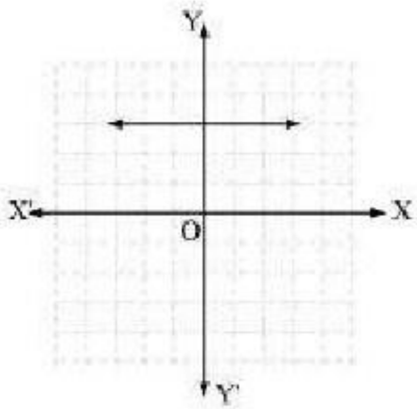
Exercise 2.1

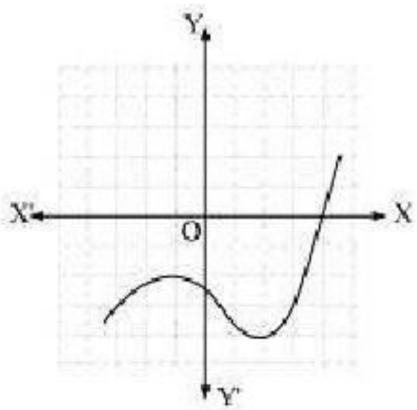
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

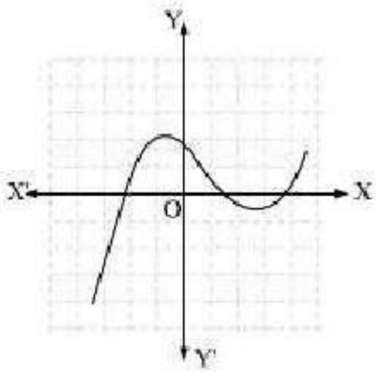
(i)



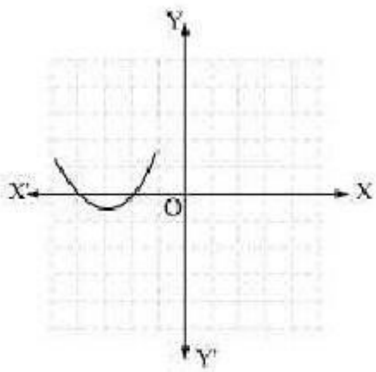
(ii)



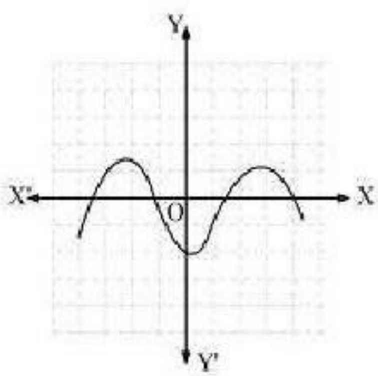
(iii)



(iv)



(v)



(v)

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2-2x-8$ $(\text{ii})\, 4s^2-4s+1$ $(\text{iii})\, 6x^2-3-7x$

$(\text{iv})\, 4u^2+8u$ $(\text{v})\, t^2-15$ $(\text{vi})\, 3x^2-x-4$

Answer:

(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when x − 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = −2

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

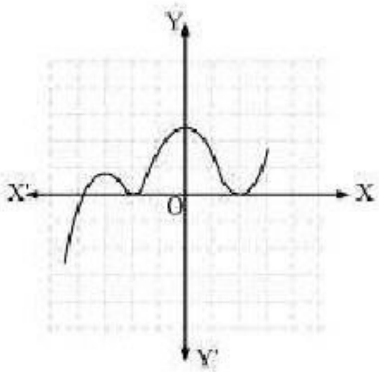
Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2$ − $4s$ + 1 is zero when $2s$ − 1 = 0, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2$ − $4s$ + 1 are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$



Answer:

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

The value of t^2 − 15 is zero when $t-\sqrt{15}=0$ or $t+\sqrt{15}=0$, i.e., when

$t=\sqrt{15}$ or $t=-\sqrt{15}$

Therefore, the zeroes of t^2 − 15 are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{-0}{1}=\frac{-\left(\text{Coefficient of } t\right)}{\left(\text{Coefficient of } t^2\right)}$

Product of zeroes = $\left(\sqrt{15}\right)\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2-x-4$

$=(3x-4)(x+1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e.,

when $x=\frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and −1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-\left(\text{Coefficient of } x\right)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) $0,\sqrt{5}$

(iv) $1,1$ (v) $-\frac{1}{4},\frac{1}{4}$ (vi) $4,1$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e.,

$x=\frac{-1}{3}$ or $x=\frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3}+\frac{3}{2}=\frac{7}{6}=\frac{-(-7)}{6}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3}\times\frac{3}{2}=\frac{-1}{2}=\frac{-3}{6}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2+8u=4u^2+8u+0$

$=4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and −2.

Sum of zeroes = $0+(-2)=-2=\frac{-(8)}{4}=\frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0\times(-2)=0=\frac{0}{4}=\frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) t^2-15

$=t^2-0t-15$

$=(t-\sqrt{15})(t+\sqrt{15})$

Answer:

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$
$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$
$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If $a = 1$, then $b = 0$, $c = \sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$
If $a = 1$, then $b = -1$, $c = 1$

Therefore, the quadratic polynomial is $x^2 - x + 1$.

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β.

$$\alpha + \beta = \frac{-1}{4} = \frac{-b}{a}$$
$$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$$
If $a = 4$, then $b = 1$, $c = 1$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) 4, 1

Let the polynomial be $ax^2 + bx + c$.

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$
If $a = 1$, then $b = -4$, $c = 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

Therefore, the quadratic polynomial is $x^2+\sqrt{5}$.

(iv) 1, 1

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
If $a=1$, then $b=-1$, $c=1$

Therefore, the quadratic polynomial is x^2-x+1.

(v) $-\frac{1}{4},\frac{1}{4}$

Let the polynomial be ax^2+bx+c, and its zeroes be α and β.

$$\alpha+\beta=\frac{-1}{4}=\frac{-b}{a}$$
$$\alpha\times\beta=\frac{1}{4}=\frac{c}{a}$$
If $a=4$, then $b=1$, $c=1$

Therefore, the quadratic polynomial is $4x^2+x+1$.

(vi) 4, 1

Let the polynomial be ax^2+bx+c.

$$\alpha+\beta=4=\frac{4}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
If $a=1$, then $b=-4$, $c=1$

Therefore, the quadratic polynomial is x^2-4x+1.

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x)=x^3-3x^2+5x-3, \qquad g(x)=x^2-2$

(ii) $p(x)=x^4-3x^2+4x+5, \qquad g(x)=x^2+1-x$

(iii) $p(x)=x^4-5x+6, \qquad g(x)=2-x^2$

Answer:

(i) $p(x)=x^3-3x^2+5x-3$

$q(x)=x^2-2$

$$
\begin{array}{r|l}
 & \quad x-3 \\
\hline
x^2-2 & x^3-3x^2+5x-3 \\
 & x^3 \qquad\quad -2x \\
 & - \qquad\qquad + \\
\hline
 & \quad -3x^2+7x-3 \\
 & \quad -3x^2 \qquad +6 \\
 & \quad + \qquad\quad - \\
\hline
 & \qquad\qquad 7x-9 \\
\hline
\end{array}
$$

Quotient = $x-3$

Remainder = $7x-9$

(ii) $p(x)=x^4-3x^2+4x+5=x^4+0.x^3-3x^2+4x+5$

$q(x)=x^2+1-x=x^2-x+1$

$$
\begin{array}{r|l}
 & x^2+x-3 \\
\hline
x^2-x+1 & x^4+0.x^3-3x^2+4x+5 \\
 & x^4-x^3+x^2 \\
 & -\quad+\quad- \\
\hline
 & x^3-4x^2+4x+5 \\
 & x^3-x^2+x \\
 & -\quad+\quad- \\
\hline
 & -3x^2+3x+5 \\
 & -3x^2+3x-3 \\
 & +\quad-\quad+ \\
\hline
 & 8 \\
\hline
\end{array}
$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2-2 \\
\hline
-x^2+2 & x^4+0.x^2-5x+6 \\
 & x^4-2x^2 \\
 & -\quad+ \\
\hline
 & 2x^2-5x+6 \\
 & 2x^2\quad-4 \\
 & -\quad+ \\
\hline
 & -5x+10 \\
\hline
\end{array}
$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2-3, 2t^4+3t^3-2t^2-9t-12$

(ii) $x^2+3x+1, 3x^4+5x^3-7x^2+2x+2$

(iii) $x^3-3x+1, x^5-4x^3+x^2+3x+1$

Answer:

(i) $t^2-3,\ 2t^4+3t^3-2t^2-9t-12$

$t^2-3 = t^2+0.t-3$

$$
\begin{array}{r|l}
 & 2t^2+3t+4 \\
\hline
t^2+0.t-3 & 2t^4+3t^3-2t^2-9t-12 \\
 & 2t^4+0.t^3-6t^2 \\
 & -\quad -\quad + \\
\hline
 & 3t^3+4t^2-9t-12 \\
 & 3t^3+0.t^2-9t \\
 & -\quad -\quad + \\
\hline
 & 4t^2+0.t-12 \\
 & 4t^2+0.t-12 \\
 & -\quad -\quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$
\begin{array}{r|l}
 & 3x^2-4x+2 \\
\hline
x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\
 & 3x^4+9x^3+3x^2 \\
 & - \quad - \quad - \\
\hline
 & \quad -4x^3-10x^2+2x+2 \\
 & \quad -4x^3-12x^2-4x \\
 & \quad + \quad + \quad + \\
\hline
 & \qquad 2x^2+6x+2 \\
 & \qquad 2x^2+6x+2 \\
\hline
 & \qquad\qquad 0 \\
\hline
\end{array}
$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$
\begin{array}{r|l}
 & x^2-1 \\
\hline
x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\
 & x^5-3x^3+x^2 \\
 & - \quad + \quad - \\
\hline
 & \quad -x^3 \qquad +3x+1 \\
 & \quad -x^3 \qquad +3x-1 \\
 & \quad + \qquad\quad - \quad + \\
\hline
 & \qquad\qquad 2 \\
\hline
\end{array}
$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Question 3:

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer:

$p(x)=3x^4+6x^3-2x^2-10x-5$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$.

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$.

$$
\begin{array}{r|l}
x^2+0.x-\frac{5}{3} & \overline{3x^4+6x^3-2x^2-10x-5} \quad (3x^2+6x+3) \\
& 3x^4+0x^3-5x^2 \\
& - \quad - \quad + \\
\hline
& 6x^3+3x^2-10x-5 \\
& 6x^3+0x^2-10x \\
& - \quad - \quad + \\
\hline
& 3x^2+0x-5 \\
& 3x^2+0x-5 \\
& - \quad - \quad + \\
\hline
& 0 \\
\hline
\end{array}
$$

$$3x^4+6x^3-2x^2-10x-5=\left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right)$$

$$=3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x+1=0$

$x=-1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x=-1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1.

Question 4:

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x-2$ and $-2x+4$, respectively. Find $g(x)$.

Answer:

$p(x)=x^3-3x^2+x+2 \qquad (\text{Dividend})$

$g(x)$ = ? (Divisor)

Quotient = (x − 2)

Remainder = (− 2x + 4)

Dividend = Divisor × Quotient + Remainder

$$x^3-3x^2+x+2=g(x)\times(x-2)+(-2x+4)$$
$$x^3-3x^2+x+2+2x-4=g(x)(x-2)$$
$$x^3-3x^2+3x-2=g(x)(x-2)$$

$g(x)$ is the quotient when we divide (x^3-3x^2+3x-2) by $(x-2)$

$$\begin{array}{r|l}
 & x^2-x+1 \\ \hline
x-2 & x^3-3x^2+3x-2 \\
 & x^3-2x^2 \\
 & -\quad + \\ \hline
 & \quad -x^2+3x-2 \\
 & \quad -x^2+2x \\
 & \quad +\quad - \\ \hline
 & \qquad\quad x-2 \\
 & \qquad\quad x-2 \\
 & \qquad\quad -\quad + \\ \hline
 & \qquad\quad 0
\end{array}$$

$\therefore g(x)=(x^2-x+1)$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x) = 0$

Answer:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with

$g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2+2x+2$ by 2.

Here, $p(x) = 6x^2+2x+2$

$g(x) = 2$

$q(x) = 3x^2+x+1$ and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$6x^2+2x+2 = 2(3x^2+x+1)$

$= 6x^2+2x+2$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$

$g(x) = x^2$

$q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$

$x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$

$g(x) = x^2$

$q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0.

Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$x^3 + 1 = (x^2) \times x + 1$

$x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3+x^2-5x+2;\quad \frac{1}{2},1,-2$

(ii) $x^3-4x^2+5x-2;\quad 2,1,1$

Answer:

(i) $p(x)=2x^3+x^2-5x+2.$

Zeroes for this polynomial are $\frac{1}{2},1,-2$

$$p\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2-5\left(\frac{1}{2}\right)+2$$
$$=\frac{1}{4}+\frac{1}{4}-\frac{5}{2}+2$$
$$=0$$
$$p(1)=2\times1^3+1^2-5\times1+2$$
$$=0$$
$$p(-2)=2(-2)^3+(-2)^2-5(-2)+2$$
$$=-16+4+10+2=0$$

Therefore, $\frac{1}{2}$, 1, and −2 are the zeroes of the given polynomial.

Comparing the given polynomial with ax^3+bx^2+cx+d, we obtain *a* = 2, *b* = 1, *c* = −5, *d* = 2

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$
$$= 8 - 16 + 10 - 2 = 0$$

$$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$$
$$= 1 - 4 + 5 - 2 = 0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain a = 1, b = −4, c = 5, d = −2.

Verification of the relationship between zeroes and coefficient of the given polynomial

Sum of zeroes $= 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1)

=2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Answer:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be $\alpha, \beta,$ and γ.

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$
$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If *a* = 1, then *b* = −2, *c* = −7, *d* = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find *a* and *b*.

Answer:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are *a* − *b*, *a* + *a* + *b*

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$p = 1$, $q = -3$, $r = 1$, $t = 1$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are $1-b, 1, 1+b$.

$$\text{Multiplication of zeroes} = 1(1-b)(1+b)$$

$$\frac{-t}{p} = 1-b^2$$

$$\frac{-1}{1} = 1-b^2$$

$$1-b^2 = -1$$

$$1+1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Question 4:

]It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer:

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x-2-\sqrt{3}\right)\left(x-2+\sqrt{3}\right) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\cline{2-2}
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\cline{2-2}
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\cline{2-2}
 & 0
\end{array}
$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or x = 7 or −5

Hence, 7 and −5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be *x* + *a*, find *k* and *a*.

Answer:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

$$
\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 26x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26-4k)x + 10 - a \\
 & (8-k)x^2 - (16-2k)x + (8k - k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x + (10 - a - 8k + k^2)
\end{array}
$$

It can be observed that $(-10+2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10+2k)$ = 0 and $(10-a-8k+k^2)$ = 0

For $(-10+2k)$ = 0,

2 k =10

And thus, k = 5

For $(10-a-8k+k^2)$ = 0

10 − a − 8 × 5 + 25 = 0

10 − a − 40 + 25 = 0

− 5 − a = 0

Therefore, a = −5

Hence, k = 5 and a = −5